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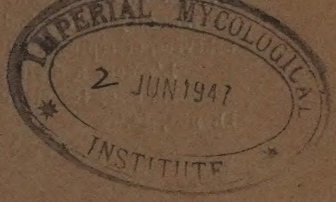

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MATHEMATICS

ON LINEAR LIE GROUPS

By A. MALCEV

(Communicated by A. N. Kolmogoroff, Member of the Academy, 25.II.1943)

E. Cartan and G. Birkhoff have pointed out some examples of Lie groups admitting of no faithful linear representation (1). On the other hand, it has been shown by Ado that every Lie algebra admits an exact matric representation (2). From this it may be inferred that connected Lie groups for which an isomorphic linear representation is possible are distinguished by a special topological structure; and to find this structure is a matter of some interest. The first result along this line has been produced by Birkhoff who was able to show that a connected nilpotent linear group is decomposed into a direct product of a compact Abelian and a simply connected nilpotent subgroups (3). The next step is the work of Cartan (4) in which it is proved that a semi-direct product of a simply connected solvable group by a connected semi-simple group of its automorphisms admits an exact linear representation*. Finally, the present author has found conditions for linear solvable groups (5). By making use of it and of the results of Cartan; it is possible to set up necessary and sufficient conditions for a given group of Lie to be linear, i. e. to admit an isomorphic continuous matric representation of a certain degree over the field of complex numbers. This is expressly the object of the present note.

Let $\mathfrak{L}$ be a semi-simple real connected Lie group admitting a faithful matric representation $\mathfrak{D}$. This representation we can choose to be real, and then the group $\mathfrak{D}(\mathfrak{L})$ uniquely determines a complex linear form $\mathfrak{L}_c$. The centrum of $\mathfrak{L}$ is contained in the centrum of $\mathfrak{L}_c$. Moreover, since the centrum of $\mathfrak{L}$ is finite, it is contained in a compact connected Abelian subgroup $\mathfrak{A}$ in $\mathfrak{L}$. The group $\mathfrak{A}$ can be included in a maximal compact subgroup $\mathfrak{B}$ of group $\mathfrak{L}_c$. Denote by $\mathfrak{Z}$ a discrete invariant subgroup $\mathfrak{L}$. Then $\mathfrak{Z} \subset \mathfrak{B}$ and $\mathfrak{B}/\mathfrak{Z}$ is a compact group and, consequently, one admitting an exact real representation $\mathfrak{D}^*$. Assigning complex values to the parameters $\mathfrak{D}^*(\mathfrak{B})$, we obtain a group $\mathfrak{B}_c^*$, which is a homomorphic image of $\mathfrak{L}_c$. This shows that the representation $\mathfrak{D}^*$ applied to the group $\mathfrak{L}$ gives a faithful representation of the group $\mathfrak{L}/\mathfrak{Z}$, and we arrive at the conclusion that all factor-groups of a semi-simple linear group are linear. Similarly, it may be proved that among locally isomorphic linear groups there exists a maximal group which is linear and all the remaining groups are its factor-groups. This universal linear group is uniquely determined also by the fact that the maximal compact subgroup of the complex form of its faithful real representation is simply connected.

* Let a group $\mathfrak{A}$ be mapped homomorphically into the group of automorphisms of a group $\mathfrak{B}$. Then by a semi-direct product of $\mathfrak{A}$ and $\mathfrak{B}$ is meant a group of pairs with the multiplication law $(a_1, b_1)(a, b) = (a_1 a, b_1^a b)$.

This settles the question as to the structure of semi-simple linear groups. In the other extreme case, viz. in that of solvable groups, the question is decided as follows. In order that a connected solvable group be linear, it is necessary and sufficient that it be a semi-direct product of its maximal compact subgroup by a simply connected invariant subgroup. The necessity of this condition was proved in (5), and the sufficiency reveals itself in proving the following general theorem.

Theorem. *In order that a connected group of Lie admit an isomorphic matric representation, it is necessary and sufficient that its radical and its maximal semi-simple subgroup separately admit a similar representation.*

Let further $\mathfrak{G}$ be a given group. Denote by $\tilde{\mathfrak{G}}$ its universal covering group. $\tilde{\mathfrak{G}}$ admits a semi-direct decomposition $\tilde{\mathfrak{G}} = \tilde{\mathfrak{L}}\tilde{\mathfrak{R}}$, where $\tilde{\mathfrak{R}}$ is the radical and $\tilde{\mathfrak{L}}$ is a maximal semi-simple subgroup. In the case of homomorphism $\tilde{\mathfrak{G}} \to \mathfrak{G}$ the radical $\tilde{\mathfrak{R}}$ is carried into the radical $\mathfrak{R}$ of the group $\mathfrak{G}$, and $\mathfrak{L}$ is mapped into a maximal semi-simple subgroup $\mathfrak{L}$ (which is, generally speaking, non-closed). The elements $l \in \mathfrak{L}$ generate automorphisms $l^{-1}\mathfrak{R}l$ of the group $\mathfrak{R}$, which are uniquely extended to the automorphism of $\tilde{\mathfrak{R}}$. Thus, $\mathfrak{L}$ is homomorphic to the group of automorphisms of $\mathfrak{L}^*$ of the radical $\tilde{\mathfrak{R}}$. According to Cartan's theorem, the semi-direct product $\mathfrak{L}^*\tilde{\mathfrak{R}}$ admits an isomorphic representation by matrices $\mathfrak{D}^*$. In virtue of the condition imposed, the group $\mathfrak{L}$ also admits an isomorphic representation $\mathfrak{D}$. Let (l, r) be an arbitrary element of $\tilde{\mathfrak{G}}$ ($l \in \tilde{\mathfrak{L}}$, $r \in \tilde{\mathfrak{R}}$). Then $\mathfrak{D}_0(l, r) = \mathfrak{D}(l) + \mathfrak{D}^*(l, r)$ is, obviously, an exact representation of the semi-direct product $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$.

We now intend to deform $\mathfrak{D}_0$ so as to obtain a representation of the semi-direct product $\mathfrak{L} \cdot \mathfrak{R}$. We have $\mathfrak{L} \cdot \mathfrak{R} \cong \mathfrak{L} \cdot \tilde{\mathfrak{R}}/Z$, $Z \subset \tilde{\mathfrak{R}}$, $\mathfrak{R} \cong \tilde{\mathfrak{R}}/Z$. $\mathfrak{R}$ is supposed to be representable. Hence, $\mathfrak{R} = \mathfrak{A} \cdot \mathfrak{S}$, $\tilde{\mathfrak{R}} = \tilde{\mathfrak{A}} \cdot \mathfrak{S}$, $Z \subset \tilde{\mathfrak{A}}$ and $\mathfrak{A} \cong \tilde{\mathfrak{A}}/Z$ is compact. $\mathfrak{S}$ can be so chosen as to be an invariant subgroup in $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$. Consequently, we have a decomposition of the group $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$ into a semi-direct product of three factors, $\mathfrak{L} \cdot \tilde{\mathfrak{A}} \cdot \mathfrak{S}$. In $\tilde{\mathfrak{A}}$ we select an independent system of one-parameter generating subgroups $A_1(t), \ldots, A_m(t)$ in such a manner that the elements $A_1(1), \ldots, A_m(1)$ generate the group Z. Let $L \in \mathfrak{L}$, $A \in \tilde{\mathfrak{A}}$, $L^{-1}AL = A_l S_l$, $S_l \in \mathfrak{S}$. The correspondence $A \to A_l$ gives an authomorphism in $\mathfrak{A}$. Since the elements $A_j(1)$ remain invariant, we have $A_l = A$ and $L^{-1}AL = AS_l$. Let us now introduce the following notion. A one-parameter matric subgroup $T(t)$ is called the centralizing path of the matrix T_0, if $T(0) = E$, $T(1) = T_0$ and $T(t)$ commutes with every matrix commuting with T_0. It may easily be found that every matrix has at least one centralizing path.

Denote by $T_j(t)$ the centralizing path of the matrix $A_j(1)$ $(j = 1, \ldots, m)$.

Let $e_{\alpha\beta}$ be the matric units of the representation $\mathfrak{D}_0$ $(\alpha, \beta = 1, \ldots, n)$ and $e_{\gamma\delta}$ $(\gamma, \delta = 1, \ldots, n+m)$ the matric units of the enclosing ring. Put

$$\bar{L} = L + \sum e_{n+j,\, n+j}, \quad \bar{S} = S + \sum e_{n+j,\, n+j},$$

$$\bar{A}_k(t) = A_k(t)\, T_k(-t) + \sum_{j \neq k} e_{n+j,\, n+j} + e_{n+k,\, n+k} \exp(2\pi i t),$$

where $L \in \mathfrak{L}$, $A_i(t) \in \tilde{\mathfrak{A}}$, $S \in \mathfrak{S}$ are matrices of the representation $\mathfrak{D}_0$ of the group $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$. The correspondence $LAS \to \bar{L}\bar{A}\bar{S}$ is a representation $\mathfrak{D}_1$ of the group $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$. The kernel of $\mathfrak{D}_1$ coincides with Z, and therefore $\mathfrak{D}_1$ is an isomorphic representation of the group $\mathfrak{L} \cdot \mathfrak{R}$. In fact, the group $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$, as regards its abstract structure, is formed by combinations of three elements (l, a, s) with the multiplication law $(l, a, s)(l_1, a_1, s_1) = (ll_1, a^{l_1}a_1, s^{l_1 a_1}s_1)$. Hence, in order to prove that $\mathfrak{D}_1$ is a representation of the group $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$, it is sufficient to verify the equalities $\bar{L}\bar{L}_1 = \overline{LL_1}$, $\bar{L}^{-1}\bar{A}\bar{L} = \overline{L^{-1}AL}$, $\bar{L}^{-1}\bar{S}\bar{L} = \overline{L^{-1}SL}$, $\bar{A}^{-1}\bar{S}\bar{A} = \overline{A^{-1}SA}$.

As $T_j(t)$ commute with every matrix of the group $\mathfrak{L} \cdot \tilde{\mathfrak{R}}$, the first, third and fourth relations are obvious, and the second follows from $L^{-1}ATL = AT \cdot S$, $\bar{L}^{-1}\bar{A}\bar{L} = \bar{A}\bar{S}$.

We have thus learned that the semi-direct product $\mathfrak{H} = \mathfrak{L} \cdot \mathfrak{R}$ admits the isomorphic representation $\mathfrak{D}_1$. The initial group $\mathfrak{G}$ is isomorphic to the factor-group of $\mathfrak{H}$ with respect to a certain central subgroup Z, and $Z \cap \mathfrak{L} = E$, $Z \cap \mathfrak{R} = E$. Suppose that the elements of Z are $L_j R_j$ $(j = 1, 2, \ldots, p)$. From the centrality of Z it follows that L_j are central in $\mathfrak{L}$, $L^{-1}R_jL = R_j$, $L_j^{-1}RL_j = R_jRR_j^{-1}$ $(L \in \mathfrak{L},\ R \in \mathfrak{R})$.

Since the centrum of $\mathfrak{L}$ is finite, Z is finite and contained in the group $\mathfrak{K} \cdot \mathfrak{R}$, where $\mathfrak{K}$ is a compact connected Abelian subgroup of $\mathfrak{L}$, containing the elements $L_1, \ldots, L_p$. Denote by $\mathfrak{Q}$ the maximal compact subgroup of the group $\mathfrak{K} \cdot \mathfrak{R}$ containing $\mathfrak{K}$, and put $\mathfrak{Q} \cap \mathfrak{R} = \mathfrak{B}$. The group $\mathfrak{B}$ is the maximal compact subgroup in $\mathfrak{R}$, and therefore $\mathfrak{R} = \mathfrak{B} \cdot \mathfrak{S}$. Choose in $\mathfrak{B}$ an independent system of one-parameter generating subgroups $B_1(t), \ldots, B_m(t)$ such that the elements $B_1(1) = R_1, \ldots, B_q(1) = R_q$ generate the whole group $\{R_j\}$. Let $T_j(t)$ be the centralizing path of the matrix $L_jR_j = L_jB_j(1)$ in $\mathfrak{D}_1$. Put

$$\bar{L} = L + \sum e_{u+j,\,u+j}, \quad \bar{S} = S + \sum e_{u+j,\,u+j},$$

$$B_k(t) = B_k(t)\,T_k(-t) + \sum_{j \neq k} e_{u+j,\,u+j} + e_{u+k,\,u+k} \exp(2\pi i t).$$

As in the previous case, we are easily convinced that the correspondence $LBS \to \bar{L}\bar{B}\bar{S}$ gives the representation of the group $\mathfrak{H}$. If $\bar{L}\bar{B}\bar{S} = \bar{E}$, then $\bar{B} = \bar{B}_1(n_1) \ldots \bar{B}_q(n_q)$, where n_j are integers. Consequently, $\bar{B} = \bar{L}_1^{-n_1} \ldots \bar{L}_q^{-n_q}$, $\bar{L}\bar{L}_1^{-n_1} \ldots \bar{L}_q^{-n_q}\bar{S} = \bar{E}$. Since the new representation is known beforehand to be faithful for the group $\mathfrak{L} \cdot \mathfrak{S}$, we have $\bar{S} = \bar{E}$, $\bar{L} = \bar{L}_1^{n_1} \ldots \bar{L}_q^{n_q}$, or $LBS \in Z$, and this representation is an isomorphic representation of the group $\mathfrak{G}$.

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MATHEMATICS

ON THE GROWTH OF HOMOGENEOUS RANDOM PROCESSES WITH INDEPENDENT SINGLE-TYPE INCREMENTS

By B. V. GNEDENKO

(Communicated by A. N. Kolmogoroff, Member of the Academy, 24.II.1943)

A homogeneous random process with independent single-type increments is a one-parametric family of random values $\xi(t)$ dependent upon one real parameter t and subjected to the three following conditions: 1) for any t_0 and t the distribution function of the quantity $\xi(t+t_0)-\xi(t_0)$ is independent of t_0; 2) the increments $\xi(t)$ during not overlapping intervals of the parameter t are independent; 3) for all t the distribution functions of the quantities $\xi(t+t_0)-\xi(t_0)$ belong to the same type *.

The full probability characteristic of this class of processes has been found by Lévy and Khintchine (3). It turned out that the function $F(x, t)$ is the distribution function of a quantity $\xi(t+t_0)-\xi(t_0)$ if and only if the logarithm of its characteristic function has for $t>0$ the form

$$\lg f(z,t)=t\left\{i\gamma z-c|z|^{\alpha}\left[1+i\beta\frac{z}{|z|}\omega(z,\alpha)\right]\right\}, \tag{1}$$

where α, β, γ, c are real constants, satisfying the conditions $0\leqslant\alpha\leqslant 2$, $|\beta|\leqslant 1$, $c\geqslant 0$, γ may take any value and

$$\omega(z,\alpha)=\begin{cases}\operatorname{tg}\frac{\pi}{2}\alpha & \text{for } \alpha\neq 1,\\ \frac{2}{\pi}\lg|z| & \text{for } \alpha=1\end{cases}$$

The constant α is called the characteristic index of the process $\xi(t)$. Throughout this work the constants α, β, γ, c retain the sense they have in formula (1).

One may essily compute by means of formula (1) that for any $t>0$ the relation

$$F(x,t)=F(xt^{-1/\alpha}-a(t),\ 1) \tag{2}$$

holds, where

$$a(t)=\begin{cases}\gamma(t-t^{1/\alpha}) & \text{for } \alpha\neq 1,\\ \frac{2}{\pi}c\beta t\lg t & \text{for } \alpha=1.\end{cases} \tag{3}$$

* According to P. Lévy and A. Khintchine the distribution functions $F_1(x)$ and $F_2(x)$ are said to belong to a common type, if with the real constants $a>0$ and b suitably chosen for all x there takes place the equation $F_2(x)=F_1(ax+b)$.

The distribution functions determined by the equation (1) are called stable. For any stable law $F(x)$, with the exception of the law of Gauss ($\alpha=2$) and of the improper laws ($\alpha=0$), there exists a constant $\theta=\theta(\alpha,\beta,c)>0$ such that [(2), p. 201; (1), Theorem 13]

$$\lim_{x\to\infty} x^{\alpha}\{1-F(x)+F(-x)\}=\theta. \tag{4}$$

Let us denote for brevity sake

$$\eta_t(t)=\xi(t+t_0)-\xi(t_0)-a(t);$$
$$F(x)=P\{\eta_t(1)<x\};$$
$$\chi(x)=1-F(x)+F(-x).$$

It follows from (2) that

$$F(xt^{-1/\alpha})=P\{\eta(t)<x\}. \tag{5}$$

The purpose of the present note is to prove the following theorems.

Theorem 1. *If a homogeneous random process $\xi(t)$ with independent increments obeys a stable law with the characteristic index $\alpha\,(0<\alpha<2)$, then, for any non-decreasing function $u(t)$*

$$\limsup_{t\to\infty}\frac{|\eta(t)|}{u(t)} \tag{6}$$

is with a probability equal to unity either zero, or infinitely great.

Proof. First suppose that the non-decreasing function $u(t)$ satisfies the condition

$$t^{-1/\alpha}u(t)\to\infty \quad \text{for } t\to\infty. \tag{7}$$

We shall show that the limit (6) is zero or infinity, according as the integral

$$\int_{\sigma}^{\infty}\frac{dt}{\{u(t)\}^{\alpha}} \tag{8}$$

converges or diverges (σ, an arbitrary positive number). The proof of this statement nearly word by word repeats the reasoning of Khintchine (4), by means of which he was able to prove that under condition (7) the limit (6) is zero if and only if the integral converges.

We have now to free ourselves from the condition (7). For that purpose we suppose that there exists a sequence $t_1<t_2<\ldots<t_n<\ldots$ ($t_n\to\infty$ for $n\to\infty$) such that for a certain constant $a<+\infty$

$$t_n^{-1/\alpha}u(t_n)<a, \quad (n=1,2,\ldots). \tag{9}$$

If the sequence $\{t_n\}$ is such that

$$\limsup_{n\to\infty}\frac{t_{n+1}}{t_{n+1}-t_n}=\infty,$$

we shall only consider a subsequence $\{t_{n_j}\}$ for which

$$\frac{t_{n_{j+1}}}{t_{n_{j+1}}-t_{n_j}}<b^{\alpha}, \quad (j=1,2,\ldots)$$

b being a certain positive constant.

We can therefore take it for granted that the sequence $\{t_n\}$ is already such that

$$\frac{t_{n+1}}{t_{n+1}-t_n}<b^{\alpha}, \quad (n=1,2,\ldots). \tag{10}$$

Denote

$$V_m = P\{|\eta(t_{m+1}) - \eta(t_m)| > N[u(t_m) + u(t_{m+1})]\},$$

where $N > 0$ is arbitrary.

In virtue of (9) and (10)

$$\frac{u(t_m) + u(t_{m+1})}{(t_{m+1} - t_m)^{1/\alpha}} \leqslant 2\frac{u(t_{m+1})}{(t_{m+1} - t_m)^{1/\alpha}} = 2\left(\frac{t_{m+1}}{t_{m+1} - t_m}\right)^{1/\alpha} t_{m+1}^{-1/\alpha} u(t_{m+1}) \leqslant 2ab;$$

therefore

$$V_m = \chi\left(N\frac{u(t_{m+1}) + u(t_m)}{(t_{m+1} - t_m)^{1/\alpha}}\right) \geqslant \chi(2abN).$$

Now, as this quantity is positive, the series $\sum_{m=1}^{\infty} V_m$ is divergent.

Let us denote here

$$w_m = P\{|\eta(t_k)| \leqslant Nu(t_k) \text{ for } s \leqslant k < m;\ |\eta(t_m)| > Nu(t_m)\}$$

and

$$U_m = P\left\{\max_{s \leqslant k \leqslant m}\left|\frac{\eta(t_k)}{u(t_k)}\right| > N\right\},$$

where s is a constant integer. It is obvious that $m > s$

$$U_m - U_{m-1} = w_m.$$

Denote by A the event consisting in that both of the inequalities

$$\left.\begin{aligned} &|\eta(t_k)| \leqslant Nu(t_k),\ (s \leqslant k \leqslant m), \\ &|\eta(t_{m+1}) - \eta(t_m)| > N[u(t_m) + u(t_{m+1})] \end{aligned}\right\} \tag{11}$$

are fulfilled. As the last inequality is independent of the previous ones, we have

$$P(A) = V_m(1 - U_m).$$

But if follows from (11) that

$$|\eta(t_{m+1})| > Nu(t_{m+1})$$

and, at the same time,

$$|\eta(t_k)| \leqslant Nu(t_k),\quad (s \leqslant k \leqslant m).$$

Denoting by B the event consisting in that all of these equations are satisfied together, we conclude from the foregoing that $A \subset B$ and, accordingly, $P(A) \leqslant P(B)$. Now

$$P(B) = w_{m+1} = U_{m+1} - U_m$$

and therefore

$$U_{m+1} - U_m \geqslant V_m(1 - U_m).$$

Hence

$$1 - U_{m+1} \leqslant (1 - U_m)(1 - V_m)$$

and so we have

$$1 - U_{m+1} \leqslant (1 - U_s)\prod_{k=s}^{m}(1 - V_k).$$

It follows from the divergence of the series $\sum V_k$ that for $m \to \infty$

$$U_m \to 1.$$

Thus, for any $s > 0$ and an arbitrary $N > 0$

$$P\left\{\max_{s \leqslant k < \infty} \frac{|\eta(t_k)|}{u(t_k)} > N\right\} = 1.$$

As it is obvious that

$$P\left\{\max_{t \leqslant \tau < \infty} \frac{|\eta(\tau)|}{u(\tau)} > N\right\} \geqslant P\left\{\max_{s \leqslant k < \infty} \frac{|\eta(t_k)|}{u(t_k)} > N\right\},$$

where s is the integer part of $t+1$, we have for any t

$$P\left\{\max_{t \leqslant \tau < \infty} \frac{|\eta(\tau)|}{u(\tau)} > N\right\} = 1$$

and consequently

$$P\left\{\limsup_{t \to \infty} \frac{|\eta(t)|}{u(t)} > N\right\} = 1.$$

N being arbitrary, this equality means that

$$P\left\{\limsup_{t \to \infty} \frac{|\eta(t)|}{u(t)} = \infty\right\} = 1.$$

On a similar reasoning we may prove the following proposition.

Theorem 2. *If a homogeneous random process $\xi(t)$ with independent increments obeys a stable law with the characteristic index $\alpha\,(0 < \alpha < 2)$, then for any monotonous function $u(t)$ $(\lim_{t \to 0} u(t) = 0)$*

$$\limsup_{t \to 0} \frac{|\eta(t)|}{u(t)}$$

is with a probability equal to unity either zero, or infinity.

A homogeneous random process $\xi(t)$ with independent increments is said to belong to the normal attraction region of the stable law $F(x)$ if with the constants $a > 0$ and $B(t)$ suitably chosen the distribution function of the random quantities

$$\frac{\xi(t) - B(t)}{at^{1/\alpha}}$$

converges to the distribution law $F(x)$. Making use of some well-known results [1], Theorem 11], one may prove the following theorem without modifying the demonstration in any way.

Theorem 3. *If a homogeneous random process with independent increment belongs to the attraction region of a stable law with the characteristic index $\alpha\,(0 < \alpha < 2)$, then whatever the monotonous function $u(t)$ may be*

$$\limsup_{t \to \infty} \frac{|\xi(t + t_0) - \xi(t_0) - B(t)|}{u(t)}$$

is zero or infinity with a probability equal to unity.

Similar results can be obtained for sums of independent random quantities.

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MATHEMATICS

THE ABSOLUTE DERIVATIVE OF FIELD OF LOCAL GEOMETRIC OBJECT IN A COMPOUND MANIFOLD

By V. WAGNER

(Communicated by S. L. Soboleff, Member of the Academy, 16. IV. 1943)

Consider an n-dimensional space X_n with the point coordinates ξ^a, which we shall call a base space. With each point ξ^a of the base X_n there is associated an m-dimensional local space $X_m(\xi^a)$ with the point coordinates u^a.

The set of all these local X_m is said to be a compound manifold and will be denoted by $X_{n+(m)}$. $X_{n+(m)}$ may be considered as $n+m$-dimensional space, whose allowable transformations of coordinates are determined by the equations

$$\xi^{a'} = \delta^{a'}_{\alpha} f^{\alpha}(\xi^{\gamma}), \quad u^{a'} = \delta^{a'}_{\alpha} \varphi^{\alpha}(\xi^{\gamma}, u^{c}), \tag{1}$$

where the local X_m are represented as m-dimensional subspaces $\xi^a = \text{const}$. We shall say that there is given in X_{n+m} a connexion of class s, if there are given point representations of the local X_m along any curve $\xi^a = \xi^a(t)$ of the class s in the base X_n, which are determined by the system of differential equations

$$\left.\begin{aligned} &\frac{du^a}{dt} + \Gamma^a(u^c, \xi^{\gamma}(t), \xi^{(k)\gamma}(t)) = 0 \\ &\left(\xi^{(k)\gamma} = \frac{d^k \xi^{\gamma}}{dt^k}\right), \quad k = 1, \ldots, s. \end{aligned}\right\} \tag{2}$$

where $\Gamma^a(u^c, \xi^{\gamma}, \xi^{(k)\gamma})$ are such functions of $\xi^{(k)\gamma}$, that the form of the equations (2) is invariant under any transformation of parameter along the curve.

A connexion of the class 1 is said to be linear if Γ^a are linear functions of $\xi^{(1)\gamma}$:

$$\Gamma^a(u^c, \xi^{\gamma}, \xi^{(1)\gamma}) = \Gamma^a_{\gamma}(u^c, \xi^{\gamma})\,\xi^{(1)\gamma}. \tag{3}$$

The representation of a local $X_m(t_0)$ on a local $X_m(t)$ is determined by the equations

$$u^a = P^a(u^c_0, t_0, t), \tag{4}$$

where the functions $P^a(u^c_0, t_0, t)$ are the general solutions of the system (2). They satisfy the following relations:

$$\left.\begin{aligned} &\frac{\partial P^a}{\partial t} + \Gamma^a(P^c, t) = 0, \quad \frac{\partial P^a}{\partial t_0} - \Gamma^b(u^c_0, t_0)\frac{\partial P^a}{\partial u^b_0} = 0 \\ &P^a(u^c_0, t_0, t_0) = u^a_0 \end{aligned}\right\} \tag{5}$$

where we denote shortly

$$\Gamma^a(u^c, t) = \Gamma^a(u^c, \xi^r(t), \xi^{(h)r}(t)).$$

We say that along a curve in the base X_n there is given a field of local geometric object, if in each local $X_m(t)$ there is given a geometric object always of the same kind. Let $\overset{(a)}{\Omega}{}^i$ $(i = 1, \ldots, N)$ be the components of a field of local geometric object of the class r(1). $\overset{(a)}{\Omega}{}^i$ are functions of point in the local X_m and of the parameter t along the curve. Under the transformations of coordinates in the local X_m determined by the second set of the equations (1) the components of the geometric object transform according to

$$\overset{(a')}{\Omega}{}^i = F^i\{\overset{(a)}{\Omega}{}^j, u^a, \varphi^a(\xi^r(t), u^c), \varphi^a_{b_1\ldots b_h}(\xi^r(t), u^c)\}, \quad h = 1 \ldots r \tag{6}$$

where $\varphi^a_{b_1\ldots b_h}$ are the partial derivatives of φ^a.

Under the point representation of the local $X_m('t)$ on the local $X_m(t)$ to the geometric object in $X_m('t)$ there corresponds a geometric object $^*\overset{(a')}{\Omega}{}^i$ in $X_m(t)$:

$$^*\overset{(a')}{\Omega}{}^i = F^i\{\overset{(a)}{\Omega}{}^j(P^j(u^c, t, 't), 't), P^i(u^c, t, 't), u^a, P^a_{b_1\ldots b_h}(P^j(u^c, t, 't), 't, t)\}. \tag{7}$$

Supposing t to be fixed and $'t$ variable, we obtain one parameter set of the geometric objects in the local $X_m(t)$. The object in the local $X_m(t)$

$$D_t\overset{(a)}{\Omega}{}^i_j = \left(\frac{\partial^*\overset{(a)}{\Omega}{}^i}{\partial' t}\right)_{'t=t} \tag{8}$$

is said to be an absolute derivative of the field $\overset{(a)}{\Omega}{}^i$.

By means of the equations (5) and (7) we obtain

$$D_t\overset{(a)}{\Omega}{}^i = \frac{\partial\overset{(a)}{\Omega}{}^i}{\partial t} - \Gamma^b\left(\frac{\partial\overset{(a)}{\Omega}{}^i}{\partial u^b} + F^i_b\right) + \sum_{b=1}^{r} F^i_{\varphi^a_{b_1\ldots b_h}}\Gamma^a_{b_1\ldots b_h} \tag{9}$$

where $F^i_{\varphi^a_{b_1\ldots b_h}}$, F^i_b are values of the partial derivatives of F^i for $\varphi^a = u^a$ and $\Gamma^a_{b_1\ldots b_h}$ are the partial derivatives of Γ^a. Under the transformation of coordinates the components of the absolute derivative transform according to the equations

$$D_t\overset{(a')}{\Omega}{}^i = F^i_{\overset{(a)}{\Omega}{}^j}\{\overset{(a)}{\Omega}{}^j, u^a, \varphi^a, \varphi^a_{b\ldots b_h}\}\, D_t\overset{(a)}{\Omega}{}^j. \tag{10}$$

It follows that only in the case where F^i are linear functions of $\overset{(a)}{\Omega}{}^i$ the object of the absolute derivative will be geometric.

A geometric object $\overset{(a)}{\underset{3}{\Omega}}{}^{i_3}$ $(i_3 = 1 \ldots N_3)$ is said to be a function of geometric objects $\overset{(a)}{\underset{1}{\Omega}}{}^{i_1}$ $(i_1 = 1 \ldots N_1)$ and $\overset{(a)}{\underset{2}{\Omega}}{}^{i_2}$ $(i_2 = 1 \ldots N_2)$, if there hold relations

$$\overset{(a)}{\underset{3}{\Omega}}{}^{i_3} = \Phi^{i_3}(\overset{(a)}{\underset{1}{\Omega}}{}^{i_1}, \overset{(a)}{\underset{2}{\Omega}}{}^{i_2}), \tag{11}$$

where the functions Φ^{i_3} do not depend on the choice of the system of coordinates. It can be shown by means of (8) that

$$D_t\overset{(a)}{\underset{3}{\Omega}}{}^{i_3} = \frac{\partial\Phi^{i_3}}{\partial\overset{(a)}{\underset{1}{\Omega}}{}^{i_1}} D_t\overset{(a)}{\underset{1}{\Omega}}{}^{i_1} + \frac{\partial\Phi^{i_3}}{\partial\overset{(a)}{\underset{2}{\Omega}}{}^{i_2}} D_t\overset{(a)}{\underset{2}{\Omega}}{}^{i_2}. \tag{12}$$

The partial derivatives $\frac{\partial \overset{(a)}{\Omega^i}}{\partial u^b}$ of the components of a geometric object $\overset{(a)}{\Omega^i}$ together with $\overset{(a)}{\Omega^i}$ form a geometric object. The absolute derivative of the field $\overset{(a)}{\Omega^i}$, $\frac{\partial \overset{(a)}{\Omega^i}}{\partial u^b}$ satisfies the relations

$$D_t \frac{\partial \overset{(a)}{\Omega^i}}{\partial u^b} = \frac{\partial}{\partial u^b} D_t \overset{(a)}{\Omega^i}. \tag{13}$$

Considering the absolute derivative of the field of the local geometric object $\overset{(a)}{\Omega^i}$, $D_t \overset{(a)}{\Omega^i}$, we obtain the object $D_t^2 \overset{(a)}{\Omega^i} = D_t D_t \overset{(a)}{\Omega^i}$, which we call an absolute derivative of $\overset{(a)}{\Omega^i}$ of the second order. Denoting by $^*\overset{(a)}{\Omega^i}$, $^*D_t \overset{(a)}{\Omega^i}$ the geometric object in the local $X_m(t)$, which corresponds to the geometric object $\overset{(a)}{\Omega^i}$, $D_t \overset{(a)}{\Omega^i}$ in the local $X_m(t)$ under its representation on $X_m(t)$, we have

$$^*D_t \overset{(a)}{\Omega^i} = \frac{\partial {}^*\overset{(a)}{\Omega^i}}{\partial t}. \tag{14}$$

It follows from this equation that

$$D^2 \overset{(a)}{\Omega^i} = \left(\frac{\partial^2 \overset{(a)}{\Omega^i}}{\partial t^2} \right)_{t=t} \tag{15}$$

and so on for the absolute derivative of any order.

A field of local geometric object is said to be constant if under the representations of the local X_m the geometric objects of the field correspond to each other. In virtue of (14) we obtain that a necessary and sufficient condition that a field of local geometric object be constant is that its absolute derivative must vanish identically.

Let $\overset{(a)}{\Omega^i}$ be a field of local geometric object defined in a region of the base X_n.

The components of the absolute derivative of $\overset{(a)}{\Omega^i}$ are functions of the linear element of the s-th order of the curve. If the connexion is linear, they are linear functions of $\xi^{(1)\gamma}$

$$D_t \overset{(a)}{\Omega^i} = D_\gamma \overset{(a)}{\Omega^i} \xi^{(1)\gamma}, \tag{16}$$

where

$$D_\gamma \overset{(a)}{\Omega^i} = \frac{\partial \overset{(a)}{\Omega^i}}{\partial \xi^\gamma} + \Gamma^b_\gamma \left(\frac{\partial \overset{(a)}{\Omega^i}}{\partial u^b} + F^i_{0\, b} \overset{(a)}{\Omega^i} \right) + \sum_{h=1}^{r} F^i_{\theta\, \gamma b_1 \ldots b_h} \Gamma^a_{\gamma b_1 \ldots b_h}. \tag{17}$$

The object $D_\gamma \overset{(a)}{\Omega^i}$ is said to be a covariant derivative of $\overset{(a)}{\Omega^i}$.

A field of a local geometric object defined in a region of the base X_n is said to be constant if it is constant along any curve. This means that its absolute derivative vanishes identically for any linear elements of the s-th order.

In the case of a linear connexion this condition is equivalent to the vanishing of the covariant derivative. Considering the representations of the local X_m along arbitrary closed curves, issuing from a point ξ^a_0, we obtain a group of point transformations in the local $X_m(\xi^a_0)$ called the group of holonomicity at this point. The group of holonomicity is a continuous Lie's group (finite or infinite).

The groups of holonomicity at different points are similar. If we have a constant field of local geometric object, then the geometric object of the field will be invariant under the transformations of the group of holonomicity.

Inversely, if a certain geometric object in the local X_m (ξ_0^α) is invariant under the transformations of the group of holonomicity, there exists a constant field of this local geometric object. In order to prove it we introduce the following system of coordinates in the local X_m. Let us join the initial point ξ_0^α with every point of the base X_m by a certain curve and represent $X_m(\xi_0^\alpha)$ on each $X_m(\xi^\alpha)$ along the corresponding curves. We choose the systems of coordinates in the local X_m in such a way that under these representations the coordinates of the corresponding points be equal. We call these systems of coordinates canonic. It can be shown that with respect to canonic coordinates the representations of the local X_m along any curve will be expressed analytically by the same equations as a certain transformation of the group of holonomicity at the point ξ_0^α. In order to obtain the constant field of the considered geometric object, we define the field whose components with respect to the canonic coordinates do not depend on ξ^α and are equal to the components of the geometric object in $X_m(\xi_0^\alpha)$.

The equations of definition of a continuous group of transformations can be considered as conditions that a certain geometric object is invariant under all of these transformations. We call this geometric object a characteristic geometric object of the group. It is easy to see that a necessary and sufficient condition that the group of holonomicity coincide with a certain group $\mathfrak{G}$ of point transformations in X_m, or with one of its subgroups, is that there exists a constant field of characteristic geometric object of the group $\mathfrak{G}$.

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ELECTRICAL ENGINEERING

A THEORY OF SERVOMECHANISMS WITH D. C. SHUNT MOTOR FEED FROM AN IONIC RECTIFIER

By V. P. NIKITIN, Member of the Academy, and N. P. KUNITZKI

In our previous analysis of the work of a follow-up drive (1) the inductance of the armature circuit of the motor of the servomechanism was not taken into consideration. In the most general case, when the speed of the driving shaft is expressed by the equation

$$n_3 = n_{0\max}\sin(\omega_0 t + \varphi_{03}) + n_{c3} + mt,$$

the following differential equations are obtained, when the inductance L of the armature circuit is allowed for:
for the angle of lag ϑ between the driving shaft and the mechanism

$$\frac{d^3\vartheta}{dt^3} + \left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\frac{d^2\vartheta}{dt^2} + \frac{1}{TB}\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{d\vartheta}{dt} + \frac{\pi a\vartheta}{30BTz_3 e\Phi} =$$
$$= \left[\frac{\pi n_{0\max}}{30}\left(\frac{1}{TB} + \frac{D}{TB\operatorname{tg}\alpha_\mu} - \omega_0^2\right)\right]\sin(\omega_0 t + \varphi_{03}) +$$
$$+ \frac{\pi n_{0\max}\omega_0}{30}\left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\cos(\omega_0 t + \varphi_{03}) + \frac{\pi}{30BT}(mt + n_{c3}) -$$
$$+ \frac{\pi}{30BT\operatorname{tg}\alpha_\mu}\left[\frac{A}{z_\mu} + D(mt + n_{c3}) + \frac{\pi m}{30}\left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\right] +$$
$$+ \frac{F\pi}{30B\operatorname{tg}\alpha_\mu z_\mu}\left(\frac{t}{T} + 1\right); \qquad (1)$$

for the speed of the mechanism

$$\frac{d^3 n}{dt^3} + \left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\frac{d^2 n}{dt^2} + \frac{1}{TB}\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{dn}{dt} + \frac{\pi a n}{30BTz_3 e\Phi} =$$
$$= \frac{\pi a}{30BTz_3 e\Phi}[n_{0\max}\sin(\omega_0 t + \varphi_{03}) + mt + n_{c3}] - \frac{F}{TB\operatorname{tg}\alpha_\mu}; \qquad (2)$$

for the current of the motor

$$\frac{d^3 i}{dt^3} + \left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\frac{d^2 i}{dt^2} + \frac{1}{TB}\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{di}{dt} + \frac{\pi a i}{30BTz_3 e\Phi} =$$
$$= \frac{\pi a D}{30Bz_3 L\operatorname{tg}\alpha_\mu}[n_{0\max}\sin(\omega_0 t + \varphi_{03}) + mt + n_{c3}] +$$
$$+ \frac{\pi a}{30Bz_3}[\omega_0 n_{0\max}\cos(\omega_0 t + \varphi_{03}) + m] + \frac{\pi a A}{30z_3 B\operatorname{tg}\alpha_\mu L} +$$
$$+ \frac{\pi a F t}{30B\operatorname{tg}\alpha_\mu L z_3} + \frac{F}{BTk\Phi}, \qquad (3)$$

where $T = \frac{L}{R}$ is the electromagnetic time constant of the motor's armature; otherwise the notation is the same as used earlier.

Solving the differential equations (1), (2) and (3), we obtain equations useful for practical computation:

$$\vartheta = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} + c_3 e^{\alpha_3 t} - \vartheta_0 \sin(\omega_0 t + \varphi_{03} - \varphi_{0\theta}) + \left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{zc\Phi mt}{a} +$$

$$+ \frac{Fz_3 c\Phi}{a \operatorname{tg}\alpha_\mu} t - \frac{30m\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)^2 z^2 c^2 \Phi^2}{\pi a^2} + \frac{zc\Phi n_{c3}}{a} + \frac{R}{ak\Phi}(Az_3 + Dn_{c3}z) +$$

$$+ \left(B + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{mzc\Phi}{a} - \frac{30\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right) Fzz_3 c^2 \Phi^2}{\pi a^2 \operatorname{tg}\alpha_\mu} + \frac{F\pi Tc\Phi z_3}{\pi a \operatorname{tg}\alpha_\mu}; \qquad (4)$$

$$n = c_1' e^{\alpha_1 t} + c_2' e^{\alpha_2 t} + c_3' e^{\alpha_3 t} + N_0 \sin(\omega_0 t + \varphi_{03} + \varphi_{0n}) + z_\mu (mt + n_{c3}) -$$

$$- \frac{30m\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right) z_\mu zc\Phi}{\pi a} - \frac{30Fzc\Phi}{\pi a \operatorname{tg}\alpha_\mu}; \qquad (5)$$

$$i = c_1'' e^{\alpha_1 t} + c_2'' e^{\alpha_2 t} + c_3'' e^{\alpha_3 t} + I_0 \sin(\omega_0 t + \varphi_{03} + \varphi_{0i}) + \frac{\Lambda}{k\Phi} +$$

$$+ \frac{Dz_\mu (mt + n_{c3})}{k\Phi} + \frac{Ft}{k\Phi} - 30\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{mDzz_\mu c}{k\pi a} -$$

$$- 30F\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right)\frac{zc}{k\pi a} + \frac{mBz_\mu c\Phi}{R} + \frac{30Fzc}{k\pi a};$$

where α_1, α_2 and α_3 are roots of the characteristic equation of the differential equations above-cited, and

$$\vartheta_0 = \frac{\pi n_{0\max}}{30(\omega_0^2 P^2 + H^2)} \times$$

$$\times \sqrt{P^2\left[H + \omega_0^2\left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\right]^2 + \omega_0^2\left[H\left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right) - P^2\right]^2};$$

$$\operatorname{tg}\varphi_{0\theta} = -\frac{\omega_0\left[H + \left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right) - P^2\right]}{P\left[H + \omega_0^2\left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\right]};$$

$$P = \omega_0^2 - \frac{1}{TB}\left(1 + \frac{D}{\operatorname{tg}\alpha_\mu}\right);$$

$$H = \frac{\pi a}{30TBzc\Phi} - \left(\frac{1}{T} + \frac{D}{B\operatorname{tg}\alpha_\mu}\right)\omega_0^2;$$

$$N_0 = \frac{\pi a n_{0\max}}{30BTz_3 c\Phi}\sqrt{\frac{1}{\omega_0^2 P^2 + H^2}}; \quad \operatorname{tg}\varphi_{0n} = \frac{\omega_0 P}{H};$$

$$I_0 = \frac{\pi a n_{0\max}}{30Lz_3(\omega_0^2 P^2 + H^2)}\sqrt{\left(\frac{DH}{B\operatorname{tg}\alpha_\mu} - \omega_0^2 P\right)^2 + \omega_0^2\left(H + \frac{DP}{B\operatorname{tg}\alpha_\mu}\right)^2};$$

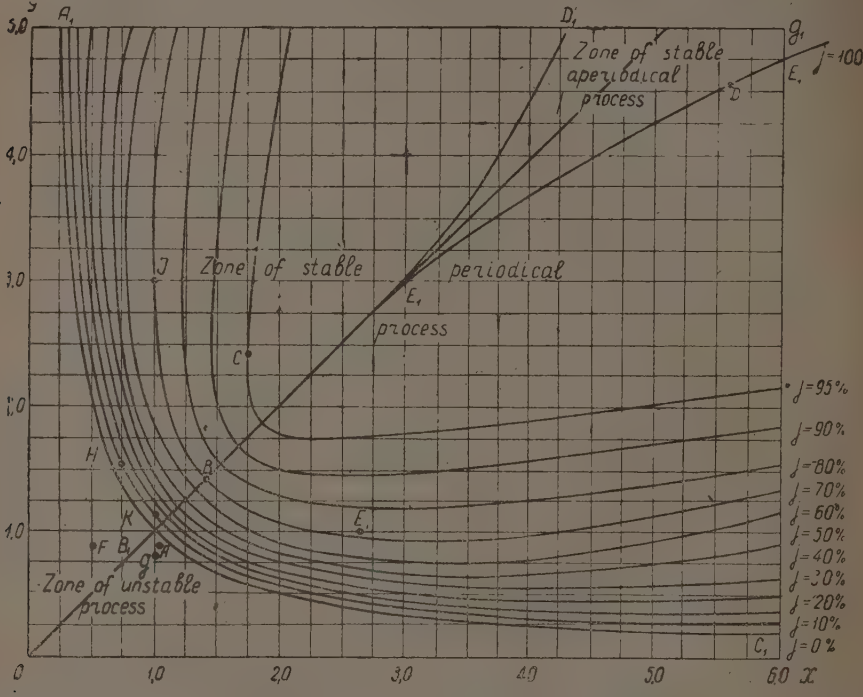
$$\operatorname{tg}\varphi_{0i} = \frac{\omega_0\left(H + \frac{DP}{B\operatorname{tg}\alpha_\mu}\right)}{\frac{DH}{B\operatorname{tg}\alpha_\mu} - \omega_0^2 P}.$$

The constants of integration in the equation (4) can be determined from the initial conditions.

Putting at zero the different parameters in (4), (5) and (6), such as $n_{0\max}$, m, F etc., we obtain the equations for the respective particular cases.

If allowance is made for the inductance of the armature circuit; the established angle of lag is, of course, expressed by the same formula as is obtained when no allowance is made for inductance (1). It is just by this angle of lag that the voltage applied to the motor of the servomechanism is determined.

In the case considered a definite angle of lag is necessary for a constant speed of rotation of the motor, in other words, for the system to be in equilibrium. For it is precisely owing to this angle of lag that, the voltage keepng up, the rotation appears on the terminals of the armature. The presence of a static angle of lag in a system of equilibrium is a very serious disadvantage with the available follow-up electric drive systems, since, on the one hand, the static angle of lag or the error



of the servomechanism should be made as small as possible; and, on the other hand, this angle is the unique cause supporting the equilibrium of the system.

Analysis of the condition for the stability of the operation of the servomechanism shows that the system will be stable if the coefficient of amplification a does not exceed a certain limit.

For a rapid determination (such as does not involve the solution of the characteristic equation of the differential equations) of the character of the transient process and of the parameters b and P of the compensating device, securing the necessary degree of damping, a diagram suggested a long time ago by Wyshnegradsky(3) and developed further by Aiserman (4) can be used. Applying this diagram to a follow-up drive, we have to plot the quantity

$$x = \frac{\frac{1}{T} + \frac{D}{B \operatorname{tg} \alpha_\mu} - \frac{p\pi}{30BT\, zc\, \Phi}}{\sqrt[3]{\frac{\pi a}{30BT\, zc\, \Phi}}}$$

on the axis of abscissae, and the quantity

$$y = \frac{\frac{1}{TB}\left(1 + \frac{D}{\operatorname{tg} \alpha_\mu} + \frac{\pi b}{30zc\, \Phi}\right)}{\sqrt[3]{\frac{\pi^2 a^2}{30^2 B^2 T^2 z^2 c^2 \Phi^2}}}$$

on the axis of ordinates.

On the diagram the zone $YA_1\ B_1\ C_1\ XO$ is the one of unstable work, and the zone $A_1\ B_1\ C_1\ F_1\ E_1\ D_1$ is that of a stable periodic process; the zone $D_1\ E_1\ F_1\ G_1$ serves for stable aperiodic process.

The curves denoted by j are lines of different damping ratio. Computing the values of x and y for some given parameters of the servomechanism, the character of the process and the degree of its damping may easily be establishe : by means of the diagram.

It is not difficult to determine the value of x and y and, consequently, of the parameters necessary for securing the desired degree of damping of the process.

Finally, by using diagram it is possible to analyse the influence of the individual parameters of the servomechanism. An increase of the amplification coefficient a transfers the process, first, from the aperiodic zone to the periodic one (when $T = 0.0035$, the process is characterized by point D for $a = 200$ and by point E for $a = 1875$); with increasing a the damping of the process is diminished, a sharp increase of a leads to an unstable work (when T is equal to 0.021, the process is characterized by point C for $a = 200$, by point B for $a = 500$, and by point A for $a = 1000$).

For the slight electromagnetic inertia of the servomechanism the process can be aperiodic (point D for $T = 0.035$); but as the inertia is increased, the process becomes periodic and the degree of damping is decreased (point C for $T = 0.024$ and point H for $T = 0.09$). A very considerable electromagnetic inertia may even lead to an unstable work (point F for $T = 0.4$).

The influence of rotary masses of the servomechanism can also be easily traced by the diagram. Increase of transmission number between the motor and the mechanism secures a more rapid damping of the process.

By means of the diagram the value of the parameter b securing the desired degree of damping can be determined. For instance, if for certain values of the servo's parameters the process is unstable at $b = 0$, and is characterized by point g, then by selecting $b = 3.87$ the process can be made stable with a damping ratio of $j = 10\%$ (point K). A further increase makes the damping grow (point I for $b = 29.3$).

The introduction of the second derivative in the composition of the voltage supplied to the motor leads, as may be easily seen from the diagram, to no increase of the damping.

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PHYSICS

ON THE INFLUENCE OF CHEMICAL BOND ON THE ASYMMETRY OF LINES OF THE EMISSION X-RAY SPECTRUM IN VARIOUS ELEMENTS

By E. E. WAINSTEIN

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1. The influence exerted by the chemical bond upon X-ray emission lines of the K-series was studied by a number of writers ([1,3,5,6]). Its existence has safely been established for light elements with ordinal numbers not exceeding

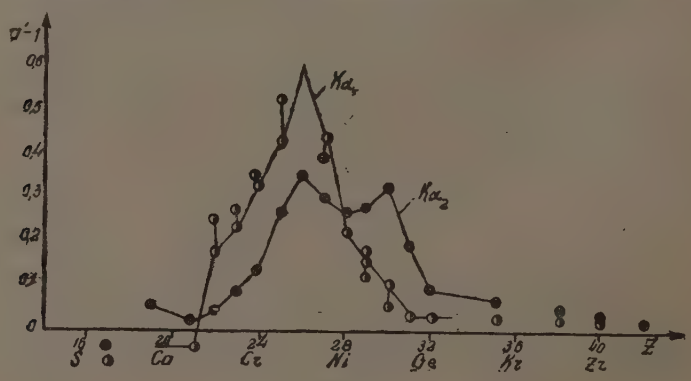

Fig. 1

30. Among the elements in which this influence is especially conspicuous we should list in the first place Na, Mg, Al, Cr, Mn, Si, S, Cl and C. The union of elements into a compound leads, as a rule, to a redistribution of intensity along the emission line (strips in light elements). It is often associated with variations in the degree of its symmetry. The lines of the $K\beta$-series of elements with $Z<19$ prove especially sensitive in this respect.

At the same time we have observed the union of atoms under investigation to be accompanied by a shift in the $K\alpha_{1,2}$ lines. The changes in the position of these lines proved, however, less important than those for the lines of the $K\beta$-series, since the appearance of $K\alpha_{1,2}$ lines of the X-ray spectrum is connected with $2p-1s$ electron transitions in the atom.

In Fig. 1 and 2 are shown experimental results of our measurements of the width and of the degree of asymmetry of the emission lines for a series of

elements from Ca to Mo*. Along the axis x are plotted the ordinal numbers of the elements, and along the axis y the deviations of the asymmetry index from unity and the half-width of the emission lines. Attention is attracted by a gradual increase in the asymmetry index of the lines $K\alpha$ as we approach the region of elements (Sc — Zn) with unfilled (deficient) $3d$ shells, as compared to the practically perfect symmetry of other elements. The index of symmetry attains its maximum value (1.6) in iron, after which it diminishes again as we approach elements with $Z>26$, so that in Zn and Ca it is practically equal to unity again. It seems natural to associate the asymmetry of the lines with the influence of the deficient structure of the $3d$ shells in the respective elements **. But this implies the allowance for the existence of an appreciable interaction between the $3d$ and $2p$ electrons of the elements of the group mentioned above; in spite of the considerable screening effect which is likely to be produced by the 8—10 electrons that separate these groups of atomic levels from one another. In case the existence of such an interaction between the peripheral electrons and those situated at a considerable distance from the atom's periphery would be established, which seems of interest from a chemical standpoint; we should find ourselves in possession of unexpected means to investigate the changes suffered by the electrons of the $3d$ shell, observed in atoms entering into compounds or alloys.

2. The magnitude of the interaction energy of $3d$—$2p$ electrons in atoms of intermediate transition elements was evaluated by means of the usual scheme of calculation of the perturbation theory of quantum mechanics. The wave-function in an imperturbed problem was written in the form:

$$\Phi_p = \frac{1}{\sqrt{2}} \left\{ \Phi_p(1)\,\Phi_d(2) - \Phi_p(2)\Phi_d(1) \right\}$$

where

$$\Phi_p(1) = \Psi_p(r_1)\,S_p(1);\quad \Phi_d(1) = \Psi_d(r_1)\,S_d(1),$$

and $\Psi_p(1)$ and $\Psi_d(1)$ are functions of coordinates of electrons in $2p$ and $3d$ state, while S_p and S_d are the corresponding spin functions.

The energy sought for will then be $U = S\tau_1\, S\tau_2\, \Phi^* \frac{e^2}{r_{12}} \Phi d\tau_1\, d\tau_2$.

As a result of summing along the spin functions; the «metathetic» interaction in the expression for U should be accounted only for electrons of $3d$ and $2p$ levels, whose spin directions coincide; the magnitude of total electrostatic repulsion does not depend on the spin direction of the interacting electrons. Thus, if the $3d$ shell of an atom lodges n_1 electrons with spins of a given direction, and n_2 electrons with spins directed in another way, then the total decrease in the gravity centre of level $2p$, associated with the interaction between the electrons it carries and the $3d$ electrons, will not be the same for the two groups of $3d$ electrons differing as to their spin-directions.

The resulting splitting of the $2p$ level in atoms with a deficient $3d$ shell will scarcely be noticeable; it will be expressed only in a widening of the lines which will be shifted towards the long-wave part of the spectrum; in the case of a different intensity of the components of the arising doublet, it will thus account for the asymmetry of the $K\alpha$ lines.

It may easily be seen that the deviation of the asymmetry index from unity; as observed for the $K\alpha$ lines, will be proportional to the number of non-coupling electrons in the $3d$ shell of the atoms; and it will not depend on the electrostatic portion of interaction between $3d$ and $2p$ electrons. The coefficient of proportionality will thereby be determined by the magnitude of the «metathetic» integral for

* In spectroscopy the term of «asymmetry index» is usually assigned to the ratio between the long-wave part of the emission line and the short-wave one, as measured at a half-height of the intensity maximum.

** The first to note the striking asymmetry of the $K\alpha_{1,2}$ lines of elements with deficient $3d$ shells was, so far as we know, Paratt[3], after him it was referred to by Snyder [2].

the $3d$—$2p$ interaction in a given element, and by the short-wave region of the curve of the line intensity; as measured at its half-height.

The magnitude of the «metathetic» integral can for simplicity sake be evaluated numerically by making use of Slater's atomic functions: $\Psi = Nr^{n^*-1}e^{-\left(\frac{Z-S}{n^*}\right)r}$.

Here N is a normalizing multiplier; found from the expression $N^2 = \frac{(2\varepsilon)^{2n^*-1}}{4\pi(2n^*)!}$; n^* is the effective quantum number for the $3d$ and $2p$ electrons; Z is the nuclear charge; S, screening constant, while $\varepsilon = \left(\frac{Z-S}{n^*}\right)$. Computing under these assumptions the magnitude of the metathetic interaction; we obtain data that are in fair agreement (with an accuracy of 3—7 %) with the general course of the change in the asymmetry index as dependent on the change of the ordinal number of atom within the group of the elements analysed. As to the order of magnitude, the data obtained coincide with the shift of the $K\alpha$ line towards the long-wave part of the spectrum.

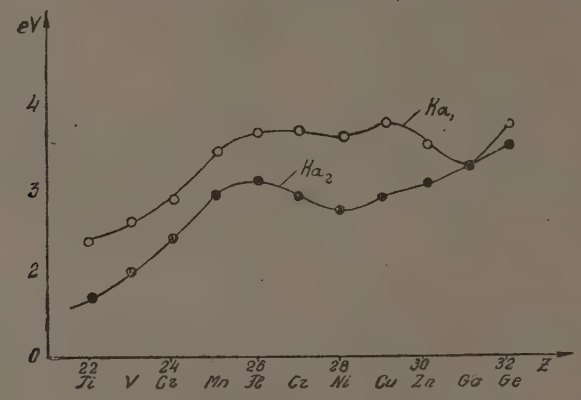

Fig. 2

An analogous calculation made for the energy of the metathetic interaction between $4s$ and $2p$ electrons gives magnitudes which are more than by two orders smaller than its value for the $3d$—$2s$ interaction.

The agreement of the possible value of the index of the line asymmetry; due to the interaction between the electrons filling the $3d$ shell and the internal $2p$ electrons; with the asymmetry index experimentally observed gives insight into the processes involving the $3d$ electrons of deficient atoms. Owing to this, the structural peculiarities of the spectra of elements belonging to this group of substances may be put into relation with the magnitudes of their magnetic susceptibility in compounds.

3. The above established relation between the asymmetry index of the $K\alpha$ lines of elements having a deficient $3d$ shell and the number of uncoupling electrons in that shell permits of an attempt to evaluate the number of the latter in a series of elements from Fe to Zn. The figure obtained appears to be in good agreement with what might be anticipated on the ground of data on the magnetism of these elements. Interesting figures have been obtained for Fe, Co and Ni. Slater, Krutter and others have shown the external $3d$ and $4s$ shells of the iron family to overlap one another. This leads to a decrease in the number of $4s$ electrons at the expense of electron density in the deficient shell of the elements listed above. According to Salter's data reinforced by magnetic measurements, it should be supposed that out of the two electrons that occupy the $4s$ shells of Fe, Co and Ni when in a free state; there remain no more than 0.6—0.7 electrons when these elements are in

metallic state. The number of $3d$ electrons in Fe; Co and Ni is thus raised up to the respective values 7.4, 8.3 and 9.3. The number of non-coupling electrons in Co is, according to our data; rather conspicuous, as compared to those relating to atoms of other transition elements. It is illustrative that the value of the asymmetry index; corresponding to the respective value obtained for metal, may be obtained when investigating oxides in the case the number of non coupling electrons in the atomic residues of the oxides coincides with the number of non-coupling electrons in atoms in a metallic state. Thus; Rosabery and Bearden [5] put the magnitude of the asymmetry index of iron in Fe_2O_3 at 1.61 (in the metal it is 1.6); for chromium in Cr_2O_3, at 1.36 (instead of 1.37 in the metal); for zinc in ZnO at 1.12 (instead of 1.12 in the metal), and so on.

It should be noted that the lines show a complete symmetry upon transition from metallic titanium ($a = 1.22$) to titanium in TiO_2; whose $3d$ shell contains no uncoupled electrons. An appreciable decrease in the asymmetry index is to be noted at the transition from metallic manganese ($a = 1.54$) to the manganese in MnO and Mn_2O_4 ($a = 1.42$). Unlike this; Snyder [6] reports that the asymmetry index of copper in $CuSO_4$ (1.26) is higher as compared to that in metallic copper (1.17); for nickel the asymmetry index rises from the metal (1.22) up to 1.89 in $NiSO_4 \cdot 7H_2O$, to drop again down to 1.70; according as the salt is gradually dissolved in water and brought to a concentration of 0.4%. The asymmetry index of cobalt increases in an analogous way from 1.4 up to the value 1.65 upon the transition from metallic state to the compound $CoCl_3 \cdot 6H_2O$.

We still lack more complete and reliable data as to the relation between the chemical bond and the asymmetry of $K\alpha$ lines. This problem stands in need for systematic investigation.

In conclusion I consider it a pleasant duty to thank I. B. Borovsky who suggested those problems to my curiosity; valuable discussions with E. L. Fainberg should also be heartily acknowledged.

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CHEMISTRY

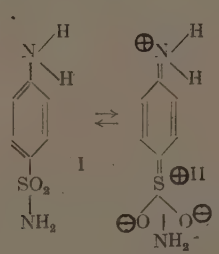
SULPHANILAMIDE COMPOUNDS AND THEIR INFLUENCE ON THE MITOGENETIC RADIATION OF BLOOD

By I. J. POSTOVSKY, A. L. JUDELES and T. I. KAZANTZEVA

(Communicated by A. E. Porai-Kochitz, Member of the Academy, 22. I. 1943)

1. Kumler and Halverstadt (1) measured the dipole moment of sulphanilamide (streptocide) (I) and found it to equal 6.63 *D*, instead of the calculated value 5.82 *D*.

On account of this difference the authors arrive at the conclusion that about 3% of sulphanilamide in the summary state of the molecule exist in the resonance structure II.



According to the authors; it is to the highly excited structure (II) that is due the chemotherapeutic activity of the preparation. The interpretation given by a number of American authors (2) to the mechanism of the action of a sulphanilamide compound (s. a. c.) is that the latter enters into competition with the essential metabolite of the bacteria for the possession of definite groups in enzymatic systems, essential for the growth and metabolism of the bacteria organism.

The general specificity of the structure and the condition of the molecule expressed by the limit resonance structure (II) makes a polar absorption possible, the latter being definitely orientated by specific active groups of the protein substrate. In this manner one or more enzymes are converted into new simplexes which do not participate in the congruent catalytic mechanism of several enzymes. In connexion with this there takes place distortion of the process of growth as well as the formation of degenerative form of the bacteria, followed by disturbances in the normal process of cell division (3), delay in multiplication, etc.

The capacity of s. a. c. to suppress the catalytic action of the enzymes can be seen in model experiments, say, in the suppression of the action of dehydrogenases of lactic, pyruvic acids (4), of the anhydrase of carbonic acid (5) and etc.

The suppression of enzymatic processes of micro-organisms (and to a certain degree of the macro-organism proper, the carrier of micro-organisms) must necessarily lead to a clear-cut stop of the emission in the short-wave ultra-violet, which is

typical of enzymatic processes, say of A. Gurvich's mitogenetic rays. The experimental checking of such a conjecture could yield support to the idea of the inhibitory effect of s. a. c. upon the enzymatic processes, and supply a new means for measuring the activity of s. a. c.

As an object for our investigations and a source of mitogenetic rays we chose blood; whose luminescence proceeds at the expense of enzymatic processes. In our experiments we followed the method suggested by A. Gurvich (6).

The s. a. c. in a concentration of 1 : 2000 served as an inductor; as a «detector» was used a 9—11-hour-old yeast culture in the form of a thin film on the surface of an agar-agar block. The exposure of the culture to blood lasted 5 minutes. The increment of the number of buds calculated as a per cent of budding cells gives an idea of the intensity of the mitogenetic radiation of blood. Deviation from + 10% to —10% points to the absence of the effect.

The results are given in Table 1.

Table 1

Influence of Sulphanilamide Compounds on the Mitogenetic Radiation of Blood

No. of run	Increment of budding yeast cells in per cent					
	Blood without s. a. c. (control)	Blood + sulphanilamide (I)	Blood + sulphapyridine (III)	Blood + sulphathiazole (IV)	Blood + 2-methylsulphanilamide (V)	Blood + N^4 acetylsulphanilamide (VI)
	1	2	3	4	5	6
1	38	2	23			
2	53	8	5			
3	34	8	8			
4	40	11	11			
5	32	15	20		29	
6	36	9	— 8		26	
7	30	15	— 2		20	
8	30	— 6	— 3	16	33	
9	41	0	21	— 2	31	32
10	38	0	— 3	6	34	27
11	51	— 3	21	— 6	30	
12	35	— 6	— 3	— 3		25
13	33			14		30
Average for compounds II	38	4				
III	38		7			
IV	38			4		
V	37				29	
VI	37					28

As it may be seen from the table, an addition to the emitting blood of chemotherapeutically active s. a. c.; viz. of sulphanilamide (I), sulphapyridine (III) and sulphathiazole (IV), leads to an extinction of luminescence in the short-wave ultra-violet, this being expressed in a distinct decrease in the number of yeast cells.

The results obtained thus furnish a new proof of the depressing effect of active s. a. c. on enzymatic processes. It should be emphasized that this effect is practically inobservable in the case of 2-methylsulphanilamide* (V) and of N^4-acetylsulphanilamide (VI). These substances are known as chemotherapeutically inactive (V) or poorly active (VI) preparations.

* 2-methylsulphanilamide has been mentioned in the paper by Tréfouel et al.[7] as an inactive preparation. No chemical data concerning this compound are given. The preparation was obtained from *o*-acetotoluidide by the same method that is used for the synthesis of sulphanilamide from acetanilide. M. p. 142—143° C. Found % N—15.28; calcd for $C_7H_{10}N_2O_2S$%—15.06.

2. When analysing the mechanism of the action of s. a. c., it is necessary to touch upon the problem of inhibition of the bacteriostatic action of s. a. c. Among inhibitors, particularly efficient is *p*-aminobenzoic acid [*p*-a. b. a.([8])]*. A single molecule of *p*-a. b. a. is sufficient to remove the bacteriostatic action of 23 000 molecules of s. a. c. ([9—10]). Thus, one is brought to suppose that the inhibition of the bacteriostatic effect is due neither to the competition of these two compounds for the possession of one and the same receptor; nor to the chemical action of these substances; but is called forth by some other reasons.

As shown above, there exists a certain relation between the bacteriostatic effect of s. a. c. and its capacity to bring about the extinction of mitogenetic rays. It seems, therefore, important to find out whether an addition of small amounts of *p*-a. b. a. to a solution of s. a. c. would lead to a restoring of mitogenetic radiation of blood.

Experiments have shown that the addition to a solution of s. a. c. (1 : 2000) of a solution of *p*-a. b. a. in a concentration 1 : 100 000 removed the depressing effect of s. a. c. from the mitogenetic radiation of blood; whose intensity then even surpassed that of the rays of the same blood in the control tests. Thus; *p*-a. b. a. is not only an inhibitor of the bacteriostatic action of s. a. c.; but also of their influence upon the mitogenetic radiation (Table 2).

Table 2

Restoration of Mitogenetic Radiation of Blood by *p*-Aminobenzoic Acid

No. of run	Increment of budding yeast cells in per cent		
	Blood without addition (control)	Blood + sulphanilamide	Blood + sulphanilamide + *p*-aminobenzoic acid
1	38	2	52
2	53	8	37
3	40	11	38
4	32	15	33
5	36	9	29
6	30	—6	40
7	41	0	44
8	51	—3	51
9	35	—6	41
10	38	0	36
Average	39	3	40

It is noteworthy that the isomeric *o*- and *m*-aminobenzoic acids are unable of restoring the mitogenetic radiation of blood, suppressed by s. a. c. It should be mentioned; moreover; that the solution of *p*-a. b. a. after a short exposure to an energetic source of mitogenetic rays becomes further capable of emitting rays, as glycocol and some other aminoacids after an exposure are capable of subsequent radiation ([11]). A solution of *p*-a. b. a. that has not been preliminarily exposed; like a solution of sulphanilamide, is incapable of radiation in the short-wave ultraviolet.

However; when a solution of *p*-a. b. a. (5 cm³) in a concentration of 1 : 100 000 or even 1 : 1 000 000**, that has not been preliminarily exposed, is added to a solution of sulphanilamide (5 cm³) in a 1 : 2000 concentration, an

* This acid is at the same time a growth hormone for certain bacteria and yeasts ([9]).
** In a maximum dilution there were about 600 molecules of sulphanilamide per 1 molecule of *p*-a. b. a.

intense mitogenetic radiation is observed at once (increment 42%; average from 8 runs).

Further investigation has to show in how far these interesting phenomena may be connected with the inhibitory effect of *p*-a. b. a.

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PHYSICAL CHEMISTRY

«ETHYLENE BANDS» IN THE EMISSION SPECTRUM OF ACETYLENE FLAME

By L. I. AVRAMENKO

(Communicated by N. N. Semenov, Member of the Academy, 5.I.1943)

From data available in literature ([1]) it is known that the following spectra are emitted by the acetylene flame: a continuous one, CH; OH and C_2.

We have been able to obtain under special conditions of a rarefied acetylene-air flame a new band spectrum, besides the known ones. 10 edges of bands have been measured.

Nos.	Wave-length λ in Å	Nos.	Wave-length λ in Å
1	3827	6	3567
2	3805	7	3540
3	3729	8	3469
4	3695	9	3377
5	3587	10	3359

There are still other feeble diffuse bands which defy measuring because of the overlapping of a continuous background and of bands of OH and CH.

Taking into consideration the diffuseness of their edges; all these bands coincide well with the so-called «ethylene bands» discovered by Vaidya ([2]). According to this author; the carrier of this spectrum is the HCO molecule. The spectra were obtained at the combustion of an acetylene-air mixture. containing 5% of acetylene.

The flame was obtained in a quartz tube, through which the combustible mixture was made to pass at a pressure of 24 mm l.g. The tube was heated by means of a nichrome oven up to 660° C.

The spectra were taken with a Zeiss quartz spectrograph with a relatively broad split (0.2 mm), the exposure being 2—4 min.

Changes in the conditions of the experiment, i. e. in the per cent of the combustible mixture and in pressure, lead either to a sharp relative weakening of the HCO spectrum or to its complete disappearance. According to our opinion, this makes it possible to consider the HCO molecule as an unstable intermediate reaction product.

The discovery of the HCO spectrum in the acetylene flame is an important step on the way to disclose the mechanism of the oxidation reaction of acetylene.

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GEOCHEMISTRY

CONTENT OF SCANDIUM IN BERYLS AND SOME OTHER MINERALS

By S. A. BOROVICK

(Communicated by V. G. Chlopin, Member of the Academy, 13.I.1943)

Scandium is the element upon which the anticipations of Mendeleyeff have been borne out with particular clearness. There is, indeed, a perfect coincidence with regard to atomic weight, specific weight of the oxide and other properties between the eka-boron; predicted by him on the basis of the periodical law, and the element discovered by Nielson in 1879 and christened by the name of scandium.

The scandium mineral, tortweitite $Sc_2Si_2O_7$ is extremely rare, but the element scandium has been detected in a great many samples, as the works of Eberhard (1) and Goldschmidt (2) have shown. Its concentration in most of these samples was not above 0.001%. There are scarcely any data on the occurrence of scandium in this Union. So far scandium has found no place among such elements as indium, germanium and beryllium which, though still known by the name of rare elements, have found very extensive application in practice. But the possibility of a practical application of scandium is not excluded either. From the geochemical point of view the data presented below are of undoubted interest.

The quantitative spectroscopic estimations of scandium were made by the arc method. Instead of an inner standard the background of the spectrogram was used. The method has been described in detail in an earlier work (3).

Below are tabulated the results of spectrum analyses of a number of beryls. The content of scandium appears to be definitely related to colour. In colourless beryls no scandium has been detected; in aquamarines scandium is nearly always present in concentrations considerable for so rare an element (0.02—0.03%). It is a fact established long since that chromium accounts for the colour of beryls. In the table are given data on the content of chromium in the beryls analysed for scandium. They bear witness in favour of what is generally recognized as the cause of colour in beryls (emeralds). The connexion between the content of scandium and the colour seems to be of secondary character.

Aside from the data given in the table, attention is invited by the presence of Ga in all samples in very insignificant concentrations; only in samples 13, 14, 17 the concentration is somewhat higher. Ni was detected in Nos. 1—12 and 28, 29; in No. 29 the concentration of Ni is rather high. To end with, a marked decrease of tin content in Nos. 27 and 28 has been found.

In conclusion it may be interesting to give data on the content of scandium in those minerals and ore samples where its concentration was found to be rather high (about several hundredth of a per cent).

1) Heavy fraction of sand separation from Mukasevo (South Urals).
2) Sample from a denudation Khverlisto (Caucasus), 0.1%.
3) Aluvial garnet (Near-Polar Urals).
4) Carbonaceous clay, Aron-Tiube.

s.	Place of sampling	Colour of beryl	Content of scandium in %	Content of chromine	Note
1	Emerald mines	Intense green	0.03	Considerable	Outer zone of zonal beryl
2	Do	Colourless	0.002	Very slight	Internal zone
3	»	Green	0.01	Considerable	
4	»	Dark-grey	0.002	Very slight	External shell of previous sample
5	»	Colourless	—	Traces	
6	»	»	—	»	Small crystals
7	»	Weakly greenish	0.002	Medium	
8	»	Greenish	0.01	Considerable	
9	»	Weakly greenish	0.005	Medium	
10	»	Coulourless	—	Traces	Small crystals
11	»	Weakly greenish	0.004	Considerable	In feldspar
12	»	Intense green	0.025	»	
13	Adun-Chilon, Sherlo Mt.	Bluish	0.02	—	Aquamarin
14	Do	Yellowish	0.18	—	»
15	»	Blue	0.02	—	»
16	»	Greenish	0.02	—	»
17	»	»	0.02	—	»
18	»	Green	0.03	—	»
19	Murzilka	White	—	—	»
20	Kyry-Bulak	Greenish	—	—	»
21	Turkestan range	Brown	—	—	»
22	Do	White	—	—	»
23	Altái	Greenish	—	—	»
24	Anonsky placer, Transbaical	White	—	—	»
25	Zavitinskoe, Chita province	Greenish	—	—	»
26	Borshchevochny ridge	Slightly pink	0.005	—	Vorobievite
27	Do	» »	0.02	—	»
28	South Africa	Green	0.02	Considerable	»
29	Do	»	0.015	«	In schist with inclusions

5) The ash of brown coal Suroshi-say (South Urals).
6) Forsterite, Kola peninsula, Ena deposit.
7) Flogopite; Kola peninsula, Ena deposit.
8) Pyroxene from gabbro-pegmatite, Mt. Siney Barancha (Urals).
9) Wolframite; wastes from plant A.

The fact that out of the great number of analysed samples (above 5 000) only a very insignificant portion showed an increased content of scandium, so did coloured beryls (emeralds) and aquamarines (all samples); is certainly worthy of note. The places where samples with increased concentration of scandium have been found deserve a more detailed investigation for the occurrence of scandium. By this means a valuable information may be obtained on the geochemistry of this element.

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GEOCHEMISTRY

A FIELD METHOD FOR COLLECTION OF GASES DISSOLVED IN NATURAL AND BORE-HOLE WATERS

By A. N. MURIN

(Communicated by V. G. Chlopin, Member of the Academy, 12.I.1943)

The customary practice with most of the gases dissolved in natural waters, when their volume and composition are to be determined, is to separate them from the water and subject afterwards to gasometric analysis.

The main field methods for the separation of gases are based on the distribution of the gas between the liquid phase and the gaseous phase. The water studied is let into an evacuated vessel, and the system liquid—gas is brought to equilibrium by shaking. After the analysis of the gaseous phase is made, one may, by using the distribution coefficients, compute the composition of the gas dissolved in the water. It should be noted; however, that the vessel in which the distillation of the gas is made, in order to be evacuated to a satisfactory extent requires either the use of a pump, securing as high a vacuum as tenths of a millimeter of Hg, or the use of an evaporizer. Both methods are inconvenient under field conditions. In our opinion; the most expedient way to extract the dissolved gas is by filling the vessel with water and by pumping then a portion of water out in order to create a vacuum.

The distilling apparatus applied by me is a most simple kind of aspirator. One of the vessels composing it (nearly 6 l in volume) is filled with water (the short tube is pinced); the other vessel (2 l in volume) is evacuated by a hand-pump of Shinz. The rubber syphon tube, connecting the vessels, is 2—3 m long. The water passes from the larger into the smaller vessel under the action of hydrostatic pressure and of the pressure of the gas that evolves from the solution. It may be remarked that a similar apparatus was used by Mogalevsky(1) for the extraction of gases from core-samples.

During the period of the extraction of gas the temperature in the vessel; as has been found by experiment, is not liable to appreciable variation; in other words, the distillation of the gas may be regarded as an isothermal process. The concentration of any of the dissolved gases, separated into gaseous phase; will be higher if the distillation process bears a quasi-static character. If; moreover; the distribution of the gas between the liquid and the gaseous phase obeys the law of Henry-Dalton; then the following equation takes place:

$$\frac{c_0}{c} = \alpha' \left(\frac{v' + \frac{v}{\alpha'}}{v} \right)^{\frac{1}{1-\alpha'}} . \qquad (1)$$

Here c_0 is the initial concentration of gas in water; c, concentration in the gaseous phase; v_0, the volume of the vessel; v; the volume of the gaseous phase; $v' = v_0 - v$, the volume of the water left over in the vessel; α'; Ostwald's coefficient of solubility (2).

When $\alpha' = 1$ (CO_2 for $t = 17°C$), the equation (1) assumes a simple form

$$\frac{c_0}{c} = e^{\frac{v}{v_0}} . \qquad (1')$$

The least amount of gas is evolved when the removal of the volume v of water from the vessel is instantaneous. This amount can be computed by means of the well-known formula

$$\frac{c_0}{c} = \frac{v}{v'} + \alpha'. \qquad (2)$$

In the second and third column of the adjoined table the values of $\frac{c_0}{c}$ are given for oxygen, nitrogen and argon; they were obtained experimentally by extracting air dissolved in water. The water was saturated with air at $t = 18°C$; the separation of gases took place at $t = 16°C$; $v_0 = 5650$ cm³, $v = 1095$ cm³. For the sake of comparison the values of $\frac{c_0}{c}$ computed by formula (1) (first column) are given together with those computed from formula (2) (fourth column)(⁴).

Table 1

Gas	$\left(\frac{c_0}{c}\right)_1$	$\left(\frac{c_0}{c}\right)_{exp}$		$\left(\frac{c_0}{c}\right)_2$	Note
O_2	0.234	0.251	0.249	0.274	Air nitrogen, i. e. one containing argon
N_2	0.220	0.230	0.226	0.257	
Ar	0.24	0.26	0.25	0.28	

The results of extraction of carbon dioxide gas dissolved in water were as follows.

Table 2

Gas	$\left(\frac{c_0}{c}\right)_1$	$\left(\frac{c_0}{c}\right)_{ex}$		$\left(\frac{c_0}{c}\right)_2$	Note
CO_2	1.52	1.55	1.60	1.61	c_0=19.8 cm³ (volume under normal conditions—1650 cm³)

In these experiments the carbon dioxide gas was extracted at $t = 11°C$ from an 0.0015 n solution of $CaCl_2$; a slight departure of α' from the respective value for pure water was not taken into consideration.

It seemed interesting in point of method to carry out the separation of gas under practically complete removal of water from the vessel. The water was extracted by portions (0.1 v_0), with a vigourous shaking of the vessel during the interval between two successive evacuations (for 3—5 minutes).

Table 3

Gas	$\left(\frac{c_0}{c}\right)_1$	$\left(\frac{c_0}{c}\right)_{exp}$		$\left(\frac{c_0}{c}\right)_2$	Note
CO_2	2.72	2.85	2.94	Infinity	c_0=80 cm³/l

It may be seen from the results of these experiments that the concentration c_0 of water dissolved gases can, in the case of a careful distillation, be computed from the experimentally determined concentrations by means of formula (1) with an accuracy of no less than 10%. For a vast majority of cases dealt with in practice this accuracy is more than sufficient.

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MINERALOGY

ON THE PROBLEM OF CLASSIFICATION AND NOMENCLATURE OF MINERALS WITHIN THE GROUP OF IRON-OXIDE HYDRATES

By B. P. KROTOV

(Communicated by V. A. Obruchev, Member of the Academy, 4.XI. 1942)

In my work of 1936 I have proposed a classification and nomenclature of minerals belonging to the group of iron oxides and their hydrates, which was based on the investigations carried out by the workers of the Academy of Sciences of the USSR and on the reviewal of literary data. These minerals were united by me in a diamorphic series:

Fe_2O_3	Fe_2O_3 aq	FeOOH	FeOOH aq.
1. Hematite	Hydrohematite	Goethite	Hydrogoethite
2. Maggemite	—	Lepidokrokite	Hydrolepidokrokite

Furthermore, along with crystalline varieties, amorphous minerals (resinous iron ore, siderite) approaching to hydrogoethite in composition have long been known to occur in nature. For them the name of ehrenwertite has been proposed by Cornu (1909), and of limonite by E. Posnjak and H. E. Merwin (1919).

In spite of a good agreement between the data obtained by several different workers on brown hematites, which revealed no 2 $Fe_2O_3.3H_2O$ compounds among the specimens studied, one still finds indications in papers and even in textbooks (e. g. in C. Doelter, Handb. d. Mineralchemie, 1926) as to the existence of natural iron oxide hydrates of different composition, without any references, however, to thermal or X-ray studies, or attempts to secure any, concerning these minerals. Nor is the nomenclature of these minerals satisfactorily developed at this juncture.

If within the group of iron oxides the terms hematite, hydrohematite and maggemite are used in the same sense by all, there is still much confusion about the group of iron oxide hydrates.

The compound Fe OOH occurs in nature in the form of two minerals: α-FeOOH and γ-FeOOH. Moreover, Weiser and Milligan (2) have established the existence of yet a third modification, β-FeOOH, yellow iron oxide, whose occurrence in nature has so far been proved by nobody. Of the two former minerals, the γ-Fe OOH was the first to be isolated. In 1789 it received the name of ruby iron mica from Becher and in 1806 was assigned the name of goethite by Lenz; in 1814 it was termed lepidokrokite by Ulmann.

The second modification, α-FeOOH, was assigned a name later, because it mainly forms microgranular and microfibrous aggregates which are found in the composition of ore bodies, and for which the name of brown hematite was proposed by Verner in 1817 instead of ironstones as they had been called by Lenz formerly.

In this ore, viz. in the geods of the brown glass head (Glocker, 1821) a ruby mica (lepidokrokite) has been encountered as outgrowths and ingrowths on the walls:

in the caverns needle-like ore (Breithaupt, 1833) and velvety blende (Philipps; 1832) have been reported. Only in very rare cases does α-FeOOH occur in the form of well-shaped crystals (Cornwallis). Hausmann as early as 1813, while describing brown hematites, pointed out the structure of a brown glass head from an aggregate of a fine-fibrous brown hematite. The term «fibrous brown hematite» was in use until the second half of the last century, when the name of goethite was introduced for this mineral, while γ-FeOOH continued under the name of ruby mica, or lepidokrokite.

Both names became rooted in science; and all attempts (by K. Willmann; 1921; Böhm, 1925, and Ramdohr, 1929) to restore the historically founded names of needle-like ore for α-FeOOH and of goethite for γ-FeOOH have been failures.

Still greater confusion surrounds α-FeOOH aq. and γ-FeOOH aq. On the one hand, the question as to the nature of water in this compound has received no final and unique solution, and this partly accounts for some indefiniteness as to the proper place of these minerals in the system, viz. it is not clear whether they have a right to an independent position as a mineral species.

As early as 1889 P. A. Zemiatchensky described a mineral from iron-ore beds of Tambov and Tula provinces (Dankovo, Lipetzk), belonging in the group of iron oxide hydrates with a content of water, ranging between 1 and $1^1/_2$ to one part of Fe_2O_3. This mineral is near in its physical properties to lepidokrokite (ruby mica) which was then still known by the historically accurate name of goethite, because its difference from goethite was merely in a certain excess of water. For that reason Zemiatchensky called it hydrogoethite, which is a very logical name.

Posnjak and Merwin from their investigations of a considerable number of specimens of brown hematite; limonite, goethite; lepidokrokite, ruby mica and xantosiderite; taken from different deposits, came to conclude that all these minerals, as regards their physical and thermal properties, belong to two mineral species: goethite and lepidokrokite. Therefore, they thought it a well established fact that no independent mineral species that could be classed as brown hematite exists in nature, and considered the latter as a goethite with adsorbed and capillary water. For a diamorphic modification of goethite they took the ruby mica (lepidokrokite).

The hydrogoethite of Zemiatchensky has not been studied by them. A number of later investigators (K. Gaubert, 1925; Böhm, 1925; Haber; 1925; Albrecht, 1926; Frebold; 1926; Hüttig, 1929; Williams and Thewlis, 1931; Schwirsch, 1933; Goldsztaub, 1935) have also recognized the identity of the lattices of the brown hematite and goethite. The excess of water against the formula of goethite was connected by some of them with the presence of adsorbed and capillary water between the fibres or with the presence of a substance in these interspaces. No ruby mica in the form of aggregates has been subjected to investigation.

The study of the nature of water in the minerals of iron oxide hydrate group; which has been carried on by N. S. Kurnakov and E. J. Rode and later by E. J. Rode alone (1926, 1930; 1936), gives ground to suppose the presence of a water of ceolytic character in the lattice of these minerals. This circumstance accounted for the preservation by these minerals of the position of mineral species in systematics.

In my work of 1936, devoted to the classification and nomenclature of minerals of the iron oxide and iron oxide hydrate groups, I have investigated the physical properties of the hydrogoethite of Zemiatchensky and referred this mineral to lepidokrokite (hydrolepidokrokite). On the other hand; provisorily, until the completion of the works started by Rode; I have preserved the position of hydrogoethite and hydrolepidokrokite in the systematics; as independent species.

At present it has become plain that Rode failed to settle the problem as to the nature of the water in hydrogoethites and hydrolepidokrokites At the same time; the works of O. Glemser (1938) give support to the opinion that a firmly bound adsorbed water is present in hydrogoethites; which the author does not believe to

enter into the lattice of the mineral. It is better, therefore, to refer hydrogoethite and hydrolepidokrokite to the number of goethite and lepidokrokite varieties and consider their position as analogous to that of chalcedony with respect to quartz.

My purpose in the above-mentioned work was to simplify the available nomenclature of iron oxide and iron oxide hydrate minerals, and, on the other hand, to clear it from obsolete and ambiguous terms. At present one step further can be made on the road towards a simplified nomenclature, by adopting, according to the suggestion of H. Himmel, 1939, the names of α-goethite and γ-goethite for the α- and γ-varieties of FeOOH, just as in the case of α- and β-quartz. This settles the question as to the historical correctness of this name for α- or γ-FeOOH. In an analogous way is to be decided the question concerning hydrogoethite and hydrolepidokroktie.

By the term of α-hydrogoethite should be described a fine-grained or fine-fibrous aggregate of α-goethite with excessive absorbed and capillary water, which forms solid and dense ochrous masses, oolites, etc. By γ-hydrogoethite (γ-FeOOH aq.) is to be described the hydrogoethite of Zemiatchensky; which is hydrolepidokrokite in reality. Should other iron oxide hydrates be found in nature, they will find their place in that classification.

As ehrenwertite should be described, according to the suggestion of Cornu, the amorphous variety of iron oxide hydrate which contains varying amounts of water, and yields goethite, hydrogoethite, or hydrohematite upon recrystallization.

The minerals of the iron oxide and iron oxide hydrate group are met with in nature not merely in the form of crystals or their aggregates, or in amorphous masses, but they also participate essentially in the composition of mineral masses and iron ores.

The mineral forming these ores is mainly one of the iron oxide or iron oxide hydrate group, but they also usually contain certain amounts of other species and varieties of this group, as well as some quantities of gangue minerals and, frequently, considerable admixtures of sulphides, carbonates, silicates and iron phosphates. The name «red hematite» was given to, and is preserved by the ores formed mainly of a dense iron oxide, which is usually to be classed with hydrohematite.

The term «brown hematite» must be retained to describe non-homogeneous masses of an iron ore formed from hydrohematite with a low admixture of hematite (hydrohematite), goethite, ehrenwertite, lepidokrokite, leptochlorite, pyrite, or siderite. Among brown hematites (ironstones) several varieties are distinguished in old literature, which are characterized by different composition, structure, aggregate state, colour, typical admixtures, etc., such are lake, marsh, bean, oolite, sod and meadow ores. The latter name was translated into Greek by Haussmann (1831) as limonite, though later on used to replace the name «brown hematite».

After the work of Posnjak and Merwin had been published; and all the specimens of brown hematite investigated by them found to contain smaller amounts of water than they should according to the formula $2Fe_2O_3.3H_2O$, there arose a necessity to give a new name to such mineral masses.

Posnjak and Merwin assigned the name of goethite to such aggregates of crystalline structure, and the name of limonite to amorphous aggregates. The same contents the term «limonite» is supposed to have by Klockmann (1912), Schwirsch (1933) and Winchell, whereas previously (1909) Cornu proposed the term «ehrenwertite».

Other authors suggest the name of brown hematite (hydrogoethite) to describe every colloidal and subcrystalline iron oxide hydrate with a content of 1—4 particles of water (A. K. Boldyrev, 1928). K. Gaubert (1925) recognized limonite for goethite.

At the same time; a number of investigators continued to apply the term limonite as a synonym of brown hematite to describe ores presenting a mixture of different mineral varieties from the iron oxide hydrate group. Part of them

are crystalline; the other amorphous with varying content of water (e. g. Galbrait, 1937).

Some of these workers place under this name different varieties of iron oxide, containing less water than one part (H. Kraus, W. Hunt, L. S. Ramsdell, 1936).

Others again mean by it only those varieties which contain more than 1 part of water (Niggli; 1926).

Because very conflicting suggestions have been made concerning what is to be described by the name «limonite», no one of them may be lawfully adopted. It seems a very expedient course to preserve the initial meaning of this term for meadow and sod ore, which is one of the different varieties of brown hematite, distinguishable by a typical structure, by certain admixtures, and usually containing different hydroxides of iron. The ores formed from a pure hydrohematite and those formed from a pure hydrogoethite (α-hydrogoethite) should be called hydrogoethite, to which name the variety of the ore—ochrous, dense, and so on—must be adjoined.

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PARASITOLOGY

NEW DETERRENT SUBSTANCES FOR PROTECTING MAN AGAINST ATTACKS OF MOSQUITOES

By D. I. BLAGOVESHCHENSKY, N. G. BREGETOVA and A. S. MONCHADSKY

(Communicated by E. N. Pavlovsky, Member of the Academy, 9.XII.1942)

The deterrent mosquito-nets proposed by E. N. Pavlovsky ([1–3]) proved to be quite efficacious as a means to keep off mosquitoes and other flying blood-suckers.

In fact, the deterrent pastes, ointments and emulsions applied to the skin are active only during a short period of time; their deterrent power is no high one, and the content of the active principle cannot be increased without irritating the skin coverings, especially if the latter are wet with transpiration. As to the appliances that are meant mechanically to keep off mosquitoes, they are cumbersome and embarrassing, not portable, and reduce both sight and hearing faculty. Being exempt of all these drawbacks, deterrent nets were successfully applied as a means against mosquitoes ([4]) and other blood-suckers ([3]). Under war conditions the necessity of similar appliances for bodies of troops of the Red Army, Navy and of frontier garrisons operating in localities infected by malaria or abounding in blood-suckers, is of paramount importance.

The difficulties of transportation and manufacture made the quest of new deterrent substances from local raw materials an urgent problem. And we undertook to solve this problem under the conditions of Central Asia.

From among a number of preparations (volatile oils and by-products obtained in the course of their purification) put at our disposal by a factory adjoined to a farm cultivating volatile oil-bearing plants near Pakhtaabad, we chose after preliminary tests the *d*-α-pinene fraction of juniperus oil. The rest of preparations either gave negative results, or were obtainable in insufficient quantities.

The juniperus oil is manufactured from young twigs of *Juniperus seravschanica* Kom., a plant which is widely distributed all over Central Asia at a height of above 2000 m ([5]). It is composed of pinene, myrrhcene and cedrol ([6,7]). Cedrol is a highly valuable medicinal preparation, whereas *d*-α-pinene, which is obtained in considerable amounts in the form of waste-products of its purification; found no application, which made its production more expensive.

We tested pinene both in a pure state and as a substitute for turpentine in deterrent mixtures suggested by Pavlovsky ([2, 3]). After preliminary experiments, our choice fell upon the two following recipes in which turpentine was substituted by *d*-α-pinene: 1) naphthalysol, 20 parts; pinene, 10 parts; water, 70 parts; 2) lysol, 5 parts; pinene, 8 parts; water, 77 parts. The procedure of preparing the mixtures and of impregnating the nets remained the same as suggested by Pavlovsky ([2,3]).

Testing of the deterrent substances for individual protection of man under natural conditions. The tests were run within August 16 and September 8 in the left-bank

flood-plain along the lower course of the Vakhsh river (Jilikul district of Stalinabad province, Tadjik SSR). For our work we chose a spot within the zone of «tugai», among dense growths of «djid», poplar and bushes. The idea of the tests implied parallel control records of the attacking mosquitoes under natural conditions, since the efficiency of the nets impregnated with the substance tested could not possibly be evaluated with a sufficient accuracy unless we could compare the number of mosquitoes attacking under identical condition a subject protected with a net and one unprotected.

The control records were made all along a 24-hour-day by the method of recording bell (8), which secures an absolute completeness and objectivity of the results, recording at the same time the main meteorological factors. Alongside of the control records, at the time of the most violent attacks of the mosquitoes, we carried out our tests of the deterrent substances. With this aim in view one person or two were made to stay next the spot of the control records, and to register the number of mosquitoe stings in the region of their face and neck which were protected with a net, previously treated with the substance to be tested. Each experiment lasted 15—30 minutes.

According to the data of 403 control records, the numeric and specific composition of the attacking mosquitoes was as follows: within one 5-minute test the average number of insects per patient was 8, the maximum being 127. Among other species there prevailed the highly active blood-sucker *Mansonia richiardii* Fic. (94%).

The rest was made up by *Anopheles superpictus* Gr. and *A. hyrcanus* Pall. (1%), and various species of the genera *Aëdes*, *Culex* and *Theobaldia* (5%). The negligible percentage of anopheles mosquitoes should not be taken to mean that they are absent in those localities, for in quarters they were as high as 70%, but rather as an evidence of their anthropophilous nature, and of their being adapted to attacks in closed rooms. We carried out a total of 59 tests with nets impregnated with the *d*-α-pinene fraction and a mixture of the pinene fraction with lysol and naphthalysol.

The results are summed up in Table 1.

Table 1

Substance under test	Total number of tests		Total duration of tests		Number of attacking mosquitoes per test	
	exper.	control	exper.	control	exper.	control
d-α-pinene	14	14	3 h.40 m.	1 h.10 m.	3	156
Naphthalysolopinene mixture	38	38	15 h.20 m.	3 h.10 m.	8	512
Lysolopinene mixture	7	7	1 h.50 m.	35 m.	1	112

A comparison of the numbers of attacking mosquitoes recorded in the experimental (direct) and control tests point to the high efficiency of the deterrent substances tested. The net is efficient during 5—6 days. The attempts to protract its efficiency by adding to the mixture 10 parts of vegetable oil proved a failure.

Tests of the deterrent substances for protecting habitations. The tests were carried out within October 1—9 in the Stalinabad district, at the Krupsky collective farm. The locality was severely infested with *Anopheles superpictus* which is the most dangerous transmitter of malaria all over Tadjikistan. The invasion of the lodgings by the mosquitoes was still enhanced by autumnal chills; the curve of the epidemy showed its usual autumnal peak.

Table 2 contains the data of several 10-minute catches made with the help of an exhauster in some of the buildings of the collective farm; these data illustrate their respective degree of infestation with mosquitoes.

Records made under natural conditions with the help of a recording bell (*) also gave exclusively *Anopheles superpictus*, but in very small numbers: during 172 records there were caught 92 mosquitoes only. In this case the adaptation of this species to attacks in a closed room rather than in the open air is borne out with sufficient clearness.

Table 2

Building	Date of record	Time of record	Number of *Anopheles superpictus*		
			Females	Males	Total
Living quarters	1.X	18.30—18.40	45	29	75
Empty stable	4.X	12.20—12.30	66	98	164
Do	6.X	15.30—15.40	84	83	167
»	9.X	12.35—12.45	86	99	185
Stable	5.X	15.20—15.30	76	29	105
»	9.X	16.35—16.45	67	10	77

The experiments were run in a habitation of a local type, a kibitka *, having clay walls, an earthen floor, and a roof made of reed. Every day, between 18.30 and 19.30, the kibitka was thoroughly freed of all mosquitoes by catching them.

During this procedure the window was screened with gauze, while for the rest of time it remained open; during the experiments it was covered with a net (with openings 1.5×1.5 cm large) treated with the solution to be tested. The door was kept closed, and the entry covered with a gauze curtain. A brief record of the tests is given below (Table 3).

Table 3

Date of record	Time of record	Number of mosquitoes	Procedure
1.X	18.30—19.30	75	Control record
2.X	18.30—19.30	19	Do
2.X	19.30	—	Window protected with a net impregnated with pinene
3.X	18.30—19.30	4	Experiment
4.X	18.30—19.30	3	Do
4.X	19.30	—	Net taken off
5.X	18.30—19.30	15	Control record
6.X	18.30—19.30	21	Do
6.X	19.30	—	Window protected with a net impregnated with a mixture of naphthalysol and pinene (No.5)
7.X	18.30—19.30	1	Experiment
8.X	18.30—19.30	1	Do
8.X	19.30	—	Net taken off
9.X	18.30—19.30	19	Control record

The results should be considered as highly satisfactory, since even those rooms whose windows were protected with a metallic net, and whose doors are sheltered with a tambour are not guaranteed from the invasion of isolated mosquitoes.

* Small hut mounted on wheels.

The *d*-α-pinene fraction of the juniperus oil is thus a highly efficient deterrent substance.

It can be applied either in a pure state, or as a substitute for turpentine in the recipes of E. N. Pavlovsky. And its practical value is especially high; because the raw material is available in large quantities and its production can be carried on on a sufficiently wide industrial scale.

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PARASITOLOGY

ON THE MOSQUITOES OF NORTH IRAN

By A. V. GUTSEVICH

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No special investigation of the mosquitoes of North Iran has as yet been carried out. Literature contains but fragmentary information on the subject and these are based on scarce and sporadic collections ([1-3]). The staff of parasitologists working in North Iran during May-July 1942 included it therefore into the program of their work.

Mosquitoes Found by the Expedition

Genera and species	Near-Caspian province (Mazanderan, Gorgan)	Iranian plateau (Shakhrud, Semnan provinces)
1. *Anopheles maculipennis* Mgn.	⊙	+
1a. » » *sacharovi* Favre	⊙	
*2. *A. hyrcanus Pall.*	⊙	
*3. » *bifurcatus* L.	+	+
*4. » *algeriensis* Theob.	+	
*5. » *plumbeus* Steph.	+	
6. » *superpictus* Grassi		+
*7. » *pulcherrimus* Theob.	+	
*8. *Uranotaenia unguiculata* Edw.	+	
9. *Theobaldia longiareolata* Mcq.	+	⊙
10. » *annulata* Schrk.		+
*11. *Mansonia richiardii* Fic.	+	
*12. *Aëdes caspius* Pall	⊙	+
*13. » *pulchritarsis* Rond.	+	+
*14. » *geniculatus* Ol.	+	
15. » *vexans* Mgn	+	
*16. *Culex hortensis* Fic.	+	+
*17. » *apicalis* Ad.	+	
*18. » *tritaeniorhynchus* Giles	⊙	
*19. » *mimeticus* Noë	+	
20. » *theileri* Theob.		+
21. » *pipiens* L.	⊙	⊙
*22. » *univittatus* Theob.		+

* The species denoted by ⊙ are abundant in the corresponding zone. Those denoted by * have for the first time been reported for the territory of Iran.

A total of 20 localities were surveyed, all of which (except the town of Tegeran) belong to four provinces: Mazanderan, Gorgan, Shakhrud and Semnan.

The collection of the expedition contains representatives of 22 species of blood-sucking mosquitoes (subfamily *Culicinae*), including 7 species of the genus *Anopheles*.

All the mosquito species found by the author in North Iran have also been reported for the USSR. 15 species, among them 5 species of *Anopheles*, are reported from Iran for the first time.

The Near-Caspian provinces (Mazanderan, Gorgan) differ materially as to the specific and numerical composition of their mosquito fauna from the provinces situated to the south of the mountain ridge Elburs (Shakhrud, Semnan). This could have been expected on account of the different geographical conditions.

The Near-Caspian provinces are endowed with a humid climate and profuse vegetation; they are well watered. The mosquito fauna is much diversified here; it is much alike that of Transcaucasia. The most numerous mosquito species are: *Aëdes caspius*, *Anopheles maculipennis*, locally *Culex tritaeniorhynchus*, *Culex pipiens* and *Anopheles hyrcanus*; the first 3 species are very active assailants of man.

In the provinces Shakhrud and Semnan (situated at an altitude of more than 1000 m above the sea level) which are characterized by their dry climate, scarcity of water basins, and poor vegetation (except in oases), the mosquito fauna approaches that of Middle Asia. Only 11 mosquito species have been detected there (as compared to 18 species found in the Near-Caspian provinces). No mass propagation of mosquitoes has been observed. The most numerous species are *Culex pipiens* and *Theobaldia longiareolata*, which are poorly active in their attacks of man.

The local population of the Near-Caspian provinces of Iran is badly infected with malaria. Thus, an inspection of 107 children employed at a factory at town Ashref (on the coast of Astrabad bay), carried out on May 3, 1942, gave the following results. The percentage of children with hyperthrophic spleens was found equal to 62, of those who contained malaria parasites in their blood, to 33 (data by V. M. Teltzov, from the Iranian expedition). The provinces Shakhrud and Semnan are infested with a local form of malaria, but the rate of infection is a much lower one.

To give an example, the inspection of children employed at a factory at town Semnan (the town is situated 200 km east of Tegeran) gave the following results; spleen index—7, parasite index—3 (data by V. M. Teltzov).

The most widely occurring and abundant among the anopheline mosquitoes of North Iran, *Anopheles maculipennis*, is, of course, the most important malaria vector.

This is especially true of the maritime regions of Mazanderan and Gorgan, where *Anopheles maculipennis* is extremely numerous everywhere and represented, alongside of the basic form, also by the subspecies *sacharovi*; the subspecies has been determined from an examination of deposited eggs. Place for the development of the aquatic stages of *Anopheles maculipennis* are offered chiefly by rice fields which occupy rather extensive areas in the Near-Caspian provinces. In the Shakhrud and Semnan provinces *Anopheles maculipennis* has nowhere been found in large numbers.

The next place as a vector of malaria belongs to *Anopheles superpictus* which was detected in districts of Semnan and Tegeran (it proved numerous only in the village Darzazin, near Semnan). It should be noticed that *Anopheles superpictus* usually appears in mass only towards the second half of the season, at the end of summer and in autumn.

The remaining 5 species of *Anopheles* found by me in Iran seem to play a secondary rôle in the transmission of malaria.

As to the abundance of mosquitoes in general (including *Culicinae* mosquitoes), the highest figures are reported for regions situated along the coast of the Astrabad

bay (Bender-Ghyaz, Bender-Shakh), where *Aëdes caspius* develops in huge numbers in the maritime swamps. During June and July, in the belt adjoining the bay, *Aëdes caspius* was conspicuously prevalent in our collections; the number of attacking mosquitoes was occasionally very high. In early June several hundreds of *Aëdes* mosquitoes could be caught at once in the maritime marshes in the vicinity of Bender-Ghyaz.

The bottom of freshly dried pools was covered with a solid crust consisting of larvae and pupae of *Aëdes caspius*.

Within the belt of rice cultivation, e. g. in the region of town Shakhi (Mazanderan), the most abundant among the *Culicinae* mosquitoes proved *Culex tritaeniorhynchus*, an active blood-sucker, the vector of the summer encephalitis in the Far East of the USSR and in Japan. The yellow fever mosquito has not been detected by the expedition.

The 2nd Epidemiological and
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HELMINTHOLOGY

FECUNDITY IN WHITE HARE AS AFFECTED BY HELMINTHIC DISEASES

By M. M. ALEYNIKOVA

(Communicated by K. I. Skrjabin, Member of the Academy, 17.IX.1942)

Data pointing to the detrimental influence of helminths upon the fecundity of animals have been known of late; in spite of its considerable practical importance, this question still remains but poorly investigated.

Theresa and Shalimov ([3]) experimentally discovered in 1937 the possibility of provoking miscarriage in white rats by injecting them with *Ascaris* toxin. Warshawsky ([2]) who was studying the fluctuation of fecundity in the Rostov marmot during 1938, has pointed out that the infestation of this animal with helminths increased the number of unpregnant females; reducing at the same time the average number of embryos per female.

The author of the present note was engaged during several years in a manysided study of parasitic worms in the white hare, and of their bearing on the numerical fluctuations of this animal. The aim of the author is to give here an analysis of the one side of the question only, viz., of the effect of helminthiasis on the fecundity of the white hare.

Our material consisted in 90 gravid white hare females caught in the Alexeyevsky and Kuznechikhinsky districts of the Tartar ASSR in 1937, 1938 and 1939.

The internal organs of the hares were subjected to helminthological dissection; carried out according to the method of K. I. Skrjabin. In order to investigate the noxious effect of the parasites on the fecundity of the hare, the statistical method has been applied.

The work consisted in computing the correlation between the intensity of the invasion and the number of embryos in the respective females. The invading helminths were: *Synthetocaulus kamenskyi* (Schulz, 1930) and *S. commutatus* (Diesing; 1851), *Dicrocoelium lanceatum* Stiles et Hassal (1896), *Trichostrongylus retortaeformis* (Zeder, 1800; Loos, 1905), *Taenia pisiformis* (Bloch, 1780).

The nature of the relation observed was illustrated by regression curves, plotted on the basis of the experimental material. We have adopted for these curves the shape of parabolas of the second order, which are the most readily computable; the general character of the relation is brought out by them with sufficient clearness.

Supposing that one of the causes responsible for the varying number of embryos in one litter may lie in the various degree of invasion, we practiced no classification according to the terms of pregnancy. Our material (a total of 113 females were examined) included no unpregnant females; we were unable to take into account those of them which participated in only one of the three litters; our data on the harmful influence of parasitic worms upon the fecundity may therefore have been undervaluated.

Partial resorption of the embryos was observed in two females caught in the Kuznechikhinsky district in 1938. It is of interest to note that both hare

females have been found to be badly infested with several species of parasitic worms simultaneously.

S y n t h e t o c a u l o s i s. A total of 86 pregnant hare females were examined for infestation with *Synthetocaulus* helminths; 36 of them have been found infested. The number of parasitic worms per one female varied from 1 to 238. The material was grouped into classes at a 10-point interval; the negative correlation obtained was a very high and fully significant one: $r = -0.6150 \pm 0.0831$.

The character of the relation established is illustrated by the corresponding regression curve (Fig. 1).

The considerable reduction in the number of embryos in those of the hare females that have been severely invaded with *Synthetocaulus*, is in agreement with the factual data on the numerical increment of the white hare as available for the Kuznechikhinsky and Alexeyevsky district of the Tartar ASSR (1).

Department	Year	Number of females examined	Average number of embryos per 1 female (data for 3 litters)	Total invasion of a given hare population with *Synthetocaulus*	
				invaded hares in %	average number of worms per 1 invaded hare
Kuznechikhinsky	1937	18	3.6	100.0	114
»	1938	4	4.0	95.4	87
»	1939	5	5.0	92.0	71
Alexeyevsky	1937	36	5.2	50.0	17
»	1938	17	4.4	64.2	51
»	1939	No data		94.4	70

The increase in the number of embryos in females from the Kuznechikhinsky district and the reduction of their number in the Alexeyevsky are to be directly related to the extinction of the synthetocaulosis observed in 1938 among hares of the former district, and to its outbreak, in the latter.

D i c r o c o e l o s i s. 90 pregnant hare females were examined for their invasion with *Dicrocoelium*. The intensity of invasion was evaluated arbitrarily

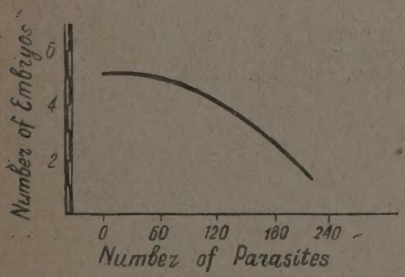

Fig. 1. *Synthetocaulus kamenskyi* Schulz, 1930. *Synthetocaulus commutatus* (Diesing, 1851)

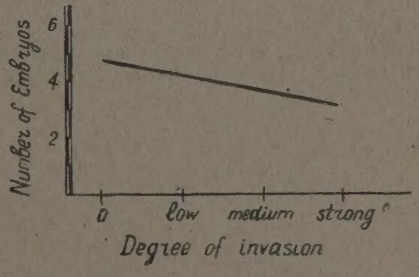

Fig. 2. *Dicrocoelium lanceatum* Stiles et Hassal, 1896

according to the following grades: absence of parasitic worms, 0, slight invasion; isolated individuals; medium invasion, presence of parasitic worms in about a half of the biliary ducts; severe invasion, presence of worms in most of the biliary ducts.

According to this classification, the bulk of the material was divided into 4 groups. The correlation coefficient obtained was: $r = 0.3170 \pm 0.09484$. This correlation being fully significant, the injurious influence of *Dicrocoelia* upon the fecundity in hare may be considered as definitely proved (Fig. 2).

Trichostrongylosis. 73 pregnant hare females were examined by us for their invasion with *Trichostrongylis*. 63 of them proved invaded. The number of *Trichostrongylis* worms per 1 female ranged within 2 — 8783 individuals.

The material was arranged into 18 classes at a 500-point interval; the correlation obtained was $r = 0.1300 \pm 0.1151$. The correlation being a rather low one, we tried and grouped our material in another way. The correlation coefficient obtained was now somewhat higher, but yet the correlation could by no means be considered as significant.

Cysticercosis pisiformis. 74 white hare females have been examined for their invasion with *Cysticercus*. To evaluate the degree of invasion, the following arbitrary standard was used: absence of paracites, 0; slight invasion, presence of solitary cystocerci, no more than 15 appreciated by a naked eye; medium invasion, from 15 to 50 cystocerci; severe invasion, above 50 individuals.

According to this classification, the whole material was subdivided into four groups. The correlation coefficient was found to be $r = 0.007 \pm 0.1352$. Thus, no definite relation between the number of embryos in the white hare and the degree of its invasion with cystocerci should be established on the ground of the material available.

We do not feel authorized, however, to refute on this ground that fecundity in hare can be affected by cystocercosis, because among our material we had only three severely infested females; as a matter of fact, the invasion of hares with these helminths is known to reach a very high degree of intensity during certain years.

The material at our disposal is insufficient to solve such question as the evaluation of a combined influence of helminths on the organism of the white hare (and the hares show, as a rule, a combined invasion with several species of parasitic worms), or to establish the relation between the various invasions, etc.

These are questions that will make the subject of our subsequent investigation, according as a more abundant material will be accumulated.

Our main task was to find out whether fecundity in hares may be affected by parasitic worms. And we believe to have solved this problem.

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